# On the Fair Pricing of Financial Structured Products: A Compensating Variation Approach.*

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**Abstract**

This paper deals with the fair pricing of financial structured products and their adequation both to investors attitudes towards risk and to rules of risk management for the issuers. First, we summarize main empirical results about the mispricing of structured financial contracts for US and various European countries. Using a geometric Brownian motion to model the risky asset dynamics with appropriate financial parameters, the average value of such mispricing lies between 3% and 6% according to product complexity. Second, we provide an explanation of such feature by introducing the notion of compensating variation. This latter concept allows to measure the monetary loss that a client or a financial institution can bear due to the utility loss of not having his best portfolio or due to significant risk exposure. We illustrate this notion for the most standard structured financial contrat, namely the Option Based Portfolio Insurance. Our results are in accordance with previous empirical studies.

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# 1 Introduction

One of the main purposes of investment banking is to develop constantly new financial instruments. This innovation process, called "financial engineering", is most often based on the use of already existing products that are combined to create new (complex) financial instruments, in order to better fit their customers'needs. One of the most prominent groups of newly introduced financial instruments resulting from such financial engineering is termed "structured products". They have been proposed to enhance portfolio returns. The demand for structured products has quickly increased.[1] They are particularly marketed through mutual funds and life insurance funds.[2]According to recent estimates, gross sales of structured retail investment products amounted to EUR 174.2bn in Europe, USD 179.8bn in the Asia-Pacific market, and to USD 65.1bn in North America during the year 2010[3]. The European retail structured products market has the top position in terms of worldwide sales. The European market is estimated to have garnered €180bn ($226bn) in gross sales in 2009, down 13% from the previous year. The total size of the North American market for structured products sold to retail investors is estimated at approximately $77bn in terms of gross sales in 2009, $61bn of which sold in the US and $9bn in Canada.

Structured products allow investors to take advantage of the risky asset rises, while being exposed only partially to market drops. The combination of basic assets gives birth to new assets with very specific characteristics whose evaluation appears very complex.[4] During the periods of financial markets decline and strong volatilities, the demand in favour of the structured products growths, in particular those with a protection clause on capital. This is the purpose of portfolio insurance that is designed to give the investor the ability to limit downside risk while allowing some participation in upside markets. Such methods allow investors to recover, at maturity, a given percentage of their initial capital, in particular in falling markets. This payoff is a function of the value at maturity of some specified portfolio of common assets, usually called the benchmark. As well-known by practitioners, specific insurance constraints on the horizon wealth must be generally satisfied. For example, a minimum level of wealth and some participation in the potential gains of the benchmark can be guaranteed. However, institutional investors for instance may require more complicated insurance contracts. The two main standard portfolio insurance strategies are the *Constant Proportion Portfolio Insurance* (CPPI) and the *Option Based Portfolio Insurance* (OBPI).[5] The CPPI has been introduced by Perold (1986) for

[1] For example, the outstanding investment on this type of fund have been estimated at a total value which borders the 500 billions of Euros (for European investors only) in 2007.

[2] With the aim of offering to their subscribers a predefined performance in any event in addition to the guarantee of initial capital, insurers use these funds to boost the bond market performance which is characterized by its relatively low yields.

[3] These data has been obtained from the database at www.structuredretailproducts.com.

[4] Discount reverse convertibles (DRC) together with reverse convertible bonds (RCB) are important examples of structured products. They are combinations of a zero bond or a coupon bond plus a short position in put options on stocks.

[5] The optimality of such dynamic portfolio strategies has been previously examined. Typ-

fixed-income instruments and Black and Jones (1987) for equity instruments (see also Perold and Sharpe, 1988). The CPPI strategy is based on a dynamic asset allocation during the whole management period. The investor begins by choosing a floor which is equal to his lowest acceptable portfolio value. Then, at any time, the amount invested on the risky asset (called the exposure) is proportional to the excess of the portfolio value over the floor (usually called the cushion). The remaining funds are invested in cash, usually T-bills. The proportional factor is defined as the multiple. Both floor and multiple are functions of the investor's risk tolerance. This portfolio insurance strategy implies that, if the cushion value converges to zero, then exposure approaches zero too. In continuous-time, this strategy prevents portfolio value from falling below the floor, except if there is a very sharp drop in the financial market before the investor can modify his portfolio weights.

The OBPI, introduced by Leland and Rubinstein (1976), is based on a static combination of a risky asset $S$ (usually a financial index such as the $S\&P$) covered by a listed put written on it. Whatever the value of $S$ at the terminal date $T$, the portfolio value is always higher than the strike $K$ of the put. The main purpose of the OBPI method is to guarantee a fixed amount only at the terminal date. In fact, if the financial market is perfect, the OBPI strategy provides a portfolio insurance at any time of the management period. The OBPI strategy is a particular case of optimal positioning. This latter one has been studied in the partial equilibrium framework by Leland (1980) and by Brennan and Solanki (1981). The portfolio value is a function of the benchmark, in a one period set up. The optimal payoff, which maximizes the expected utility, depends typically on the risk aversion of the investor. Carr and Madan (2001) consider markets in which exist out-of-the-money european puts and calls of all strikes, which allows to study the optimal positioning in a complete market. This is the counterpart of the assumption of continuous-time trading.[6] Such type of portfolio insurance strategy corresponds to optimal portfolio strategies, under specific assumptions, as shown by Bertrand *et al.* (2001).[7] The optimal payoff (maximizing the expected utility) depends crucially on the risk aversion and prudence of the investor (see e.g. Eeckhoudt and Gollier, 2005; Bertrand and Prigent, 2010).

However, two main problems arise: First, investment banks have to determine the costs of issuing such a given structured product. This is typically done

ically, the investor is assumed to maximize the expected utility of his terminal wealth, by trading in continuous time (see Cox and Huang, 1989; Cvitanic and Karatzas, 1996). The continuous-time setup is also usually considered to study portfolio insurance (see for example, Grossman and Vila, 1989, Basak, 1995, and Grossman and Zhou, 1996). The main hypothesis is that markets are complete: all portfolio profiles at maturity are perfectly hedgeable.

[6] This approximation is justified when there is a large number of option strikes (eg. for the S&P500, for example). Due to practical constraints, liquidity, transaction costs..., portfolios are in fact discretely rebalanced.

[7] Note that, in continuous-time, El Karoui, Jeanblanc and Lacoste (2005) prove that, under a fixed guarantee at maturity, the Option Based Portfolio Strategy (OBPI) is optimal for quite general utility functions (see also Jensen and Sorensen (2001) for a particular case).

by using standard arbitrage-theoretical tools, assuming that financial markets are perfect from active management of these financial institutions. Second, customer's potential utility gains induced by buying a given financial structured product would be evaluated.

For the first pricing problem, note that structured financial products consist of two or more different components, one of which must be a derivative (see Stoimenov and Wilkens, 2005). They are issued by financial institutions such as banks and are intended to private or institutional investors. They can be traded on an organized exchange market or sold directly by their issuing institution which will quote bid and ask prices. Since structured products are built up of simpler components, the valuation methodology usually breaks them down into their integral parts, *i.e.* simpler financial instruments. This approach should ease the analysis and pricing of each component. The portfolio made up of these simpler instruments must have the same payoff profile as the structured product. Given the absence of arbitrage opportunities in financial markets, the value of the structured product is equal to the sum of these individual components. This approach allows the use of simple model to calculate fair market prices for the simpler products. When it is not possible to decompose a product into simple components, that is when the structured product is a combination of complex instruments which are difficult to valuate, numerical procedures have to be used to valuate the product (Monte Carlo experiments). In this paper, we do not consider the issue of how the issuer replicates and hedges the payoffs of the fund. It is known that most of the time asset swaps are involved in such replication strategies. Moreover, there might be possible issue of conflict of interest between asset management subsidiary and the derivative trading desk when these two parties enter into an asset swap. For instance, the Financial Stability Board notes for Exchange Trade Funds (ETF): "As there is no requirement for the collateral composition to match the assets of the tracked index, the synthetic ETF creation process may be driven by the possibility for the bank to raise funding against an illiquid portfolio that cannot otherwise be financed in the repo market. In case of unexpected liquidity demand from ETF investors, the provider might face difficulties liquidating the collateral and may be faced with the difficult choice of either suspending redemptions or maintaining them and facing a liquidity shortfall at the bank level. In short, risks increase if the bank considers the synthetic ETF structure as a stable and inexpensive source of funding for illiquid securities. ETF investors may not always have sufficient control over collateral arrangements to enable them to prevent such a situation." What is true for ETF structuring is also true for other structured products such as those studied here.

For the second problem about the utility gains provided by the structured products, the risk aversion is indeed crucial to describe the investors behavior. As mentioned by Breuer and Perst (2007), "structured products are combinations of derivatives and underlying financial assets which exhibit structures with special risk/return profiles that may not be otherwise attainable on the capital market without significant transaction costs being incurred – at least for

private investors (see, e.g., Das, 2000)." Therefore, investors may agree to pay an (implicit) additional cost to have access to such products. Taking account of the investors psychology, of their cognitive biases and emotional reactions, behavioral finance can also provide a specific framework for the study of the optimal positioning of these products. Several studies have been published on this research topic. For instance, Hens and Riger (2008) show that the investor will include more complex structured products than standard equities in his portfolio. Driessen and Maenhout (2005) examine optimal positioning problems, assuming either expected utility or the CPT of Tversky and Kahneman (1992). Pfiffelmann (2008) show how some specific structured products such as capital linked notes depend crucially on the given reference level. Ben Ameur and Prigent (2010), Jin and Zhou (2008), and Prigent (2008) examine portfolio optimization with rank dependent expected utility. In this framework, it can be proved for example that, under some specific assumptions about financial and risk attitude parameters, portfolio profiles such as straddles can be optimal. In this paper, our main concern is to examine of on the main explanations of the mispricing of financial structured products, which is always observed empirically. Indeed, the misevaluation can be justified by the risk taken by the issuer (the financial institution that sells the product) while the customer (who buys the product) can bear an utility loss from not having his best portfoli profile. To deal with such problem, we introduce the notion of compensating variation which allows the measure of the willingness to pay for having a product that fits better preferences than more standard ones.

This paper is organized as follows. Section 2 summarizes main previous results about the fair pricing of financial structured products. Section 3 recalls the definition and properties of the most standard structured product, namely the standard option based portfolio insurance. Usual mispricing values of this product are detailed. Section 4 introduces the notion of compensating variation which is applied in various frameworks to calibrate the mispricing levels. In a first step, we consider an investor who has not a direct access to the financial derivatives market. We determine the compensating variation with respect to the true optimal payoff with derivatives that can be provided by the bank. It can be related to costs that the client may accept to benefit from such financial service. In a second step, we assume that the banker provides exactly the optimal payoff with insurance constraint of the customer. Therefore, the banker may bear an utility loss for not having his own optimal portfolio profile. This can potentially explain the mispricing of the structured products.

# 2 Empirical findings about fair pricing of structured products

There are relatively few empirical studies about fair pricing of structured products. The usual way to assess the fairness of these products is first to determine the payoff structure of the product and then, using standard financial formulas, to price them in a Black-Scholes world, which means specifically that the risky reference asset dynamics is described by a geometric Brownian motion. Thus, the theoretical fair value of the product at inception can be compared with its issue price.

The first studies devoted to structured products were conducted on the US markets. The most earlier empirical studies on European structured products markets focus on Switzerland, Germany and the Netherlands. The adopted approach consists in comparing prices in the primary or secondary market to theoretical fair values.

## 2.1 The case of the US market

Some authors have analyzed the pricing of convex products from the US markets, among them are Chen and Kensinger (1990) and Chen and Sears (1990) who examine the pricing of convex instruments on the S&P 500 and find substantially positive and negative price deviations from the theoretical values.

Chen and Kensinger (1990) analyze the market for market-index certificates of deposit (MICDs). These products are variable-rate certificates of deposit for which the interest rate is contingent upon the performance of the S&P 500 index but with a guaranteed minimum interest rate. By comparing the implied standard deviations form options written on the S&P 500 index with that of the option component of the MICD, the authors are able to show that there exist significant differences between theoretical values and issuers price. Chen and Sears (1990) examine a similar product, the S&P 500 500 indexed note (termed "SPIN"). The SPIN is a combination of a bond and a call option on the S&P 500 index. Using averages of implied volatilites as well as their long term historical values, they show evidence that these products can be over-valued or under-valued. Baubonis et al. (1993) consider equity-linked certificates of deposit issued by Citicorp. They find that an overpricing ranging from 2.5% to 4%. Hernandez *et al.* (2011) examine reverse exchangeable securities. A reverse exchangeable bond is a bond which pays a relatively high fixed rate but the "principal payment" on the maturity is a contingent claim – contingent upon the price of a pre-specified asset on the maturity date of the bond. If the closing price of the underlying asset is higher than the predetermined price, the investors will receive the "face value" of the bond, just like a traditional bond. However, if the closing price of the underlying asset is below the predetermined exercise price, the investors will receive a fixed number of shares of the underlying asset. They also find empirical evidence of an over pricing of these products (see also Benet et al., 2006; Chen and Wu, 2007). Although

the existing litterature documents some overpricing of structured products, it is only recently that some authors take a more negative view of these products. Henderson and Pearson (2010) call structured products the 'dark side of financial innovation' and argue that investors would be better on the money market than to buy the structured product they analyze.

## 2.2 The case of the German and Swiss market

The retail structured product market is well established in Switzerland and Germany with a large and active volume of products traded. This is reflected in the high number of research papers devoted to these markets. Wilkens et al. (2003) propose the first empirical study on financial structured products sold in the German market. They analyze a large number of reverse convertibles (or equity-linked notes without principal protection) and of discount certifcates during the exchange trade of November 2001. By extracting implied volatility from call option traded on the European exchange, they are able to determine the theoretical values of these products and to compare them with that of the secondary market. Their study reveals an "overpricing" of both reverse convertibles and discount certificates. Stoimenov and Wilkens (2005) investigate fair pricing of equity-linked structured products in the German private retail banking sector. They compare product prices with theoretical (fair) values using prices of exchange-traded options. Their results show that, in the primary market, all types of equity-linked structured products are, on average, priced above their theoretical values. In general, more complex products incorporate higher implicit premiums. In the secondary market, the overpricing decreases as the products approach maturity (*i.e.* the life cycle hypothesis). At issuance, what they call "classic" structured products[8] on DAX index sell at an average of 2.06% above their theoretical values. Entrop *et al.* (2009) analyze open-end leverage certificates on the German market. This new generation of certificates exhibits two main innovations: the issuers announce a price-setting formula according to which they are willing to buy and sell the certificates over time and the product have a potentially perpetual lifetime. Again, they find that the price-setting formula implies a relatively high profit potential for issuers. They show that their results are consistent with the "life cycle hypothesis" for structured financial products.

Wasserfallen and Schenk (1996) investigate a sample of 13 capital protected products in the Swiss market. They document a significant pricing bias in favor of the issuing institution. Burth *et al* (2001) analyze securitized covered call writing (concave) strategies in Switzerland through the initial pricing of 275 products. Their study covers all plain vanilla concave products on Swiss blue chips that were outstanding on August 1, 1999. They find a significant aver-

[8] These products have concave payoffs and are basically the combination of a long postilion in a zero coupon bond and a short position in a Put written on the DAX index. Equivalently, they can be analyzed as a combination of a long position in the underlying and of a short position in a Call. The investor buys the underlying asset at a discount but gives up a substantial part of the upside potential.

age overpricing of 1.91% in the primary market. They find structured products with a coupon payment to be substantially more mispriced than those without a coupon (3.22% *vs.* 1.40%). Grünbichler and Wohlwend (2005) examine the pricing of 192 structured products without a capital guarantee. They compare the implied volatilities of the options contained in the structured products with those of comparable EUREX options. They illustrate that there inefficiencies do exist on the Swiss market for structured products. Wallmeier and Diethelm (2008) examine the market pricing of Swiss exotic structured products and conclude also to a significant overpricing.

## 2.3 The case of other European countries

More recently, Jørgensen et al. (2011) study principal protected notes (PPNs) issued in the Danish retail market. By comparing actual offer prices with theoretical fair values, they find that PPNs are overpriced by about 6%. More importantly, they show that only half of the overpricing can be explained by the costs disclosed by sellers at the time of issuance. Thus, there exists an unexplained cost component (*i.e.* hidden costs). Other empirical studies on the fair pricing of structured products have been conducted in Sweden by Frohm (2008) who evaluate twenty two index-linked notes from January 2005 to January 2007 that represent about 40% of the structured products issues in Sweden during 2005. He does not observe any significant mispricing with respect to the secondary market. On the contrary, Szymanowska *et al.*(2009) evaluate the reverse convertibles in the Netherlands. They conclude that the mispricing is about 6%.

In France, most structured products known as "fonds à formule[9]" are of the convex payoff type and do not pay any coupon during their lifetime. One of the key features of these "fonds à formule" is that no formal secondary market exists and even the issuer bank has no formal obligation to redeem them. If redemption is allowed, the investor may loose all the benefits of the formula.[10] Investors can also have access to structured products on Euronext through Warrants and Certificates[11] which are not mutual funds but are listed on the stock exchange and tradable like shares. On December 31, 2010, the outstanding of funds with a formula amounted to 61.8 billion euros, that is 4.62% of total outstanding of French funds. Bertrand and Prigent (2011b) shows that the mispricing lies between 2% and 5%, depending on product complexity.

[9] This expression could be translated by "funds with a formula" because a mathematical formula is involved.

[10] It is explicitly stated that "any redemption of shares before the maturity date will be carried out at a price which will depend on market parameters that day" (after deduction of redemption fees). It may be very different (higher or lower than) from the amount obtained by applying the announced formula.

[11] Following their launch on the French market in 1989, the number of available products rose to 3,700 by the end of 2000. Five years later, that figure had almost doubled to 6,358 warrants and certificates from around ten issuers. More than 20 institutions are now active in this market, which currently accounts for an annual transaction volume in excess of 30 billion euros and offers a choice of over 12,000 products.

# 3 The standard financial structured product

In what follows, we analyse the fair pricing problem of the standard structured fund, introduced by Leland and Rubinstein (1976). This product belongs to the large familly of option based portfolio insurance funds. It consists of a static combination of a risky asset $S$ which is usually a financial index such as the $S\&P$ 500 and of a put written on it. Whatever the financial market fluctuations, the portfolio value is always above the strike $K$ of the put, which provides a guarantee at maturity corresponding to a fixed amount. We compute the fund fair value to gauge the potential mispricing.[12]

## 3.1 Capped OBPI

This type of insurance strategy is the standard one. It is a (small) generalization of the OBPI by introducing the following condition: from a given value of the underlying asset, the payoff is constant (see Bertrand and Prigent (2011a) for more details about such structured product). For the French market, we find an average fair value of $-2.4\%$ for such standard products in the sample, for usual volatility levels. To illustrate the methodology, we consider the following example correponding to a fund issued in France.[13]

### 3.1.1 Fund Fair Pricing

- After the 8 years (denoted $T$) of the management period, the initial portfolio value $V_0$ is guaranteed: $V_T \geq V_0$,
- This value is increased by the performance of the Eurostoxx 50 index, computed since inception. However, this performance is capped at 100%.

Additionally, in exchange of the dividends received by the fund, the investor obtains at maturity the capital guarantee as well as the capped performance of the Eurostoxx 50 index. At maturity, the payoff of this fund is the following:

- $V_0$ if $S_T < S_0$,
- $V_0 \frac{S_T}{S_0}$ if $S_0 \leq S_T < 2S_0$,
- $2V_0$ if $S_T \geq 2S_0$.

**Proposition 1** *At inception, the arbitrage value of this fund, $V_0$, is given by:*[14] $V_0(S_0) = V_0 e^{-rT} + \frac{V_0}{S_0}\left[Call\left(T, S_0, S_0, r, \sigma\right) - Call\left(T, S_0, 2S_0, r, \sigma\right)\right]$

Figure (1) displays the payoff of this fund.

[12] We do not take account of a term structure of volatilities but sometime, if it is useful, we take account of the interest rate term structure.

[13] Fund '"Allianz Efficicio" issued by Allianz.

[14] See Appendix 1.

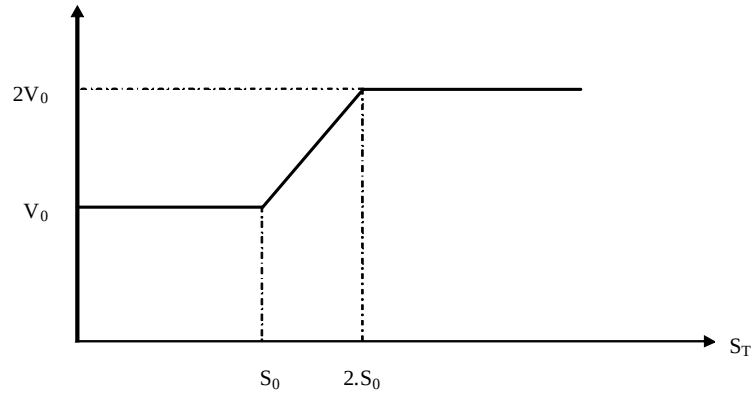



Figure 1: Payoff of the capped OBPI

#### 3.1.2 Numerical Example

At inception, the continuous-time zero-coupon interest rate with maturity 8 years is $r = 3,84\%$. The values of the parameters for the risky index are $S_0 = 100$ and $\sigma = 25\%$. Thus, we obtain the following initial fund fair value: $V_0(S_0) = 97,61$. Note that, if the volatility is lower (resp. higher) and equal to 20% (resp. 30%) for instance, the initial fund value is $V_0(S_0) = 98,92$ (resp. $96,16$). As shown in Table (1), the fund value is decreasing in volatility. However, the fair pricing is close to the benchmark value. For example, for a standard volatility level $\sigma = 20\%$, it is only equal to 98.92% ($-1.08\%$ of the benchmark value equal to 100).

Table 1: Fund Values as function of volatility

| | | **Fund Value** |
|---|---|---|
| | **15%** | **99.93** |
| | **20%** | **98.92** |
| **Volatility** | **25%** | **97.61** |
| | **30%** | **96.16** |
| | **35%** | **94.64** |

## 3.2 Truncated OBPI

We analyze now a left truncated OBPI. It is equal to a standard OBPI, except that it does not provide the same guarantee level for underlying asset values smaller than a given value. There exists a step in the portfolio profile. We find an average fair value of $-3.3\%$ for such products in the French sample (for usual volatility values).

### 3.2.1 Fund Fair Pricing

To illustrate this case, we examine the following fund:[15]

- After the 5 years (denoted $T$) of the management period, the initial portfolio value $V_0$ is guaranteed: $V_T \geq V_0$.
- This value is increased by 75% of the quarterly average performance, computed since inception, of the DJ Eurostoxx 50 Index (without dividends) denoted $S$ hereafter:
  - If $\frac{S_{i/4}}{S_0} - 1 > +15\%$ $(i = 1, ..., 20)$ then $\frac{V_{i/4}}{V_0} - 1 = \frac{S_{i/4}}{S_0} - 1$,
  - If $-15\% \leq \frac{S_{i/4}}{S_0} - 1 \leq +15\%$ $(i = 1, ..., 20)$ then $\frac{V_{i/4}}{V_0} - 1 = +15\%$,
  - If $\frac{S_{i/4}}{S_0} - 1 < -15\%$ $(i = 1, ..., 20)$ then $\frac{V_{i/4}}{V_0} - 1 = 0\%$.

Thus, the quarterly virtual payoff of this fund is given by:

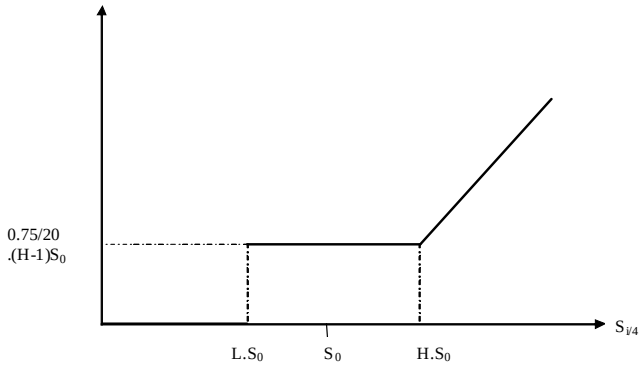
$$\begin{cases} 0.75\frac{1}{20}\left((H-1)V_0 + (S_{i/4} - HS_0)^+\frac{V_0}{S_0}\right), i = 1, ..., 20 & if\ S_{i/4} \geq L.S_0, \\ 0 & else. \end{cases}$$

Figure (2) displays the features of the quarterly performance of this fund with $L = 0.85$ and $H = 1.15$.

We use the term "virtual payoff" because these payoffs are not received by the investors at the time they occur but rather at maturity. Thus, these payoffs occur at time $i/4$ and do not earn any interest to the investor until maturity. Therefore, we must account for this and discount these payoffs over the time period $[i/4, T]$. At maturity, the initial portfolio value is guaranteed meaning that an amount equal to $V_0 e^{-rT}$ must be invested in the riskless asset at the riskfree rate, $r$ (assumed constant over time). Additionally, in exchange of the dividends received by the fund, the investor obtains an insurance on his initial investment value.

[15] Fund AMAREO issued by Société Générale.

Figure 2: Quarterly Return on Truncated OBPI




**Proposition 2** *At inception, the arbitrage value of this fund, $V_0$, is:*[16]

$$V_0(S_0) = V_0 e^{-rT}$$

$$+\frac{1}{4T}\sum_{i=1}^{4T} e^{-r(T-\frac{i}{4})} e^{-r\frac{i}{4}} \mathbb{E}_{\mathbb{Q}}\left[\mathbb{I}_{[S_{i/4}\geq L.S_0]} 0.75\left\{(H-1)V_0 + (S_{i/4} - HS_0)^+ \frac{V_0}{S_0}\right\}\right],$$

*which is equivalent to:*

$$V_0(S_0) = V_0 e^{-rT} + \frac{0.75}{4T} V_0 \times \tag{1}$$

$$\sum_{i=1}^{4T} e^{-r(T-\frac{i}{4})}\left[e^{-r\frac{i}{4}}(H-1)\left[1 - N\left(d\left(i/4, L\right)\right)\right] + Call\left(i/4, S_0, HS_0, r, \sigma\right)\frac{1}{S_0}\right].$$

*with:* $d\left(i/4, L\right) = \frac{Ln(L) - \left(r - \frac{\sigma^2}{2}\right)\frac{i}{4}}{\sigma\sqrt{i/4}}$.

### 3.2.2 Numerical Example

The following values are used for the financial market parameters: $\sigma = 25\%$ and $r = 4.55\%$[17]. The fund has a time to maturity of 5 years and is such that $L = 0.85$ and $H = 1.15$. Table 2 displays the Black-Scholes values of the 20 quarterly payoffs that entered into this structured product.

[16] See Appendix 2.

[17] This is the continuous time interest rate corresponding to the discrete one, $r = 4.65\%$.

Table 2: BS Value of the 20 Calls at inception

| **Time to maturity** | **Payoff** | **Time to maturity** | **Payoff** |
|---|---|---|---|
| 0,25 | 0.4387 | 2,75 | 0.8429 |
| 0,50 | 0.4596 | 3,00 | 0.8878 |
| 0,75 | 0.4949 | 3,25 | 0.9328 |
| 1,00 | 0.5353 | 3,50 | 0.9780 |
| 1,25 | 0.5777 | 3,75 | 1.0234 |
| 1,50 | 0.6211 | 4,00 | 1.0691 |
| 1,75 | 0.6650 | 4,25 | 1.1149 |
| 2,00 | 0.7093 | 4,50 | 1.1610 |
| 2,25 | 0.7537 | 4,75 | 1.2074 |
| 2,50 | 0.7982 | 5,00 | 1.2541 |
| | | TOTAL | **16.52** |

Moreover, $V_0 e^{-rT} = 79.672$. Thus $V_0(S_0) = 96,197$. Thus, if the fund is sold \$100 about \$3.80 are taken from investors. Notice that front-end sales loads are already paid by investors. Additionally, we can consider that the management fees correspond to the dividends received by the fund but not transferred to investors. But recall that our pricing methodology assumes a perfect market. Thus, what would be a plausible impact of the inclusion of various market imperfections on the fund value? Are the \$3.80 a credible amount?

In Table 3, the fund values are displayed when both the volatility and the riskfree rate are allowed to vary[18].

Table 3: Fund Values as function of volatility and riskfree rate

| | | **Volatility** | | | | |
|---|---|---|---|---|---|---|
| | | **15.00%** | **20.00%** | **25.00%** | **30.00%** | **35.00%** |
| | **3.00%** | 99.32 | 100.59 | 102.06 | 103.66 | 105.33 |
| **Riskfree** | **3.50%** | 97.52 | 98.78 | 100.22 | 101.79 | 103.43 |
| | **4.00%** | 95.77 | 97.02 | 98.43 | 99.97 | 101.58 |
| **Rate** | **4.50%** | 94.08 | 95.31 | 96.70 | 98.21 | 99.79 |
| | **5.00%** | 92.44 | 93.66 | 95.03 | 96.50 | 98.05 |

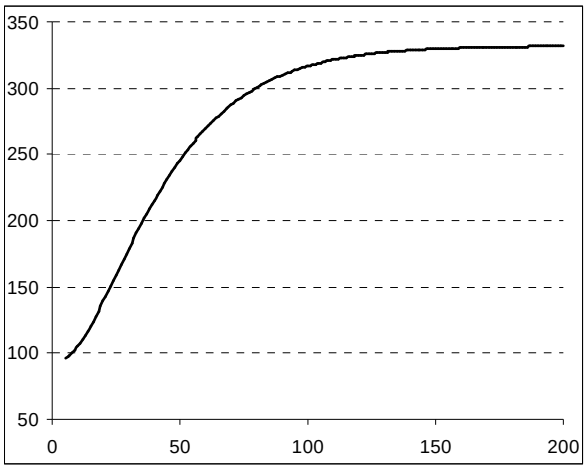
In Table 4, the fund values are displayed when the time to maturity increases. The base case for other parameter values is used.

Figure (3) shows that the fund value tends to a limit as maturity increases. Notice that the same shape would be obtained with the volatility.

Table 4: Fund Values as function of time to maturity

| **Time to maturity** | **Fund Value** | **Time to maturity** | **Fund Value** |
|---|---|---|---|
| **5** | 96,20 | **13** | 113,60 |
| **6** | 97,17 | **14** | 116,89 |
| **7** | 98,57 | **15** | 120,33 |
| **8** | 100,35 | **16** | 123,90 |
| **9** | 102,47 | **17** | 127,58 |
| **10** | 104,89 | **18** | 131,35 |
| **11** | 107,57 | **19** | 135,20 |
| **12** | 110,49 | **20** | 139,11 |

Figure 3: Fund Value as a function of Maturity




It is interesting to notice that other kinds of structuring leads to about the same pricing. For instance, asset management firms sell funds which offer a performance over a 5 years period equal to the average of the quarterly performance (if positive) of a reference index. In this case, the initial value of this fund can be determined.

**Proposition 3** *At inception, the arbitrage value of this fund, $V_0$, is given by:*

$$\begin{aligned} V_0(S_0) &= V_0 e^{-rT} + \frac{1}{4T}\sum_{i=1}^{4T} e^{-r(T-\frac{i}{4})} e^{-r\frac{i}{4}} \mathbb{E}_{\mathbb{Q}}\left[(S_{i/4}-S_0)^+\right] \\ &= V_0 e^{-rT} + \frac{1}{4T}\sum_{i=1}^{4T} e^{-r(T-\frac{i}{4})} Call\left(i/4, S_0, HS_0, r, \sigma\right) \qquad (2) \end{aligned}$$

[18] In Table 3, we report the discrete time parameter values for the interest rate.

# 4 Compensating Variations

In this section, we introduce the quantitative index of investor's satisfaction introduced by de Palma and Prigent (2008, 2009) to measure the utility loss when the optimal portfolio of the client is not available on the market (see also Anderson *et al.* 2008). It is also applied to measure the utility loss of the financial institution that bears the risk when issuing the product It is based on the standard economic concept of *compensating variation.* If an investor with risk aversion $\gamma$ and initial investment $V_0$ can buy his optimal portfolio, his expected utility is $\mathbb{E}[U_\gamma(V_T^*); V_0]$. If this investor selects an optimal portfolio among only those available, then he will get the expected utility $\mathbb{E}[U_\gamma(V_T^{*(\lambda)}); V_0]$. He will get the same expected utility provided that he invests an initial amount $\tilde{V_0} \geq V_0$. Therefore, this investor requires a compensation $\tilde{V_0}$ which satisfies:

$$\mathbb{E}[U_\gamma(V_T^*); V_0] = \mathbb{E}[U_\gamma(V_T^{*(\lambda)}); \tilde{V_0}]. \tag{3}$$

The amount $\tilde{V_0}$ is in line with the certainty equivalent concept in expected utility analysis. It can be viewed as an "implied initial investment" necessary to maintain the level of expected utility. The same analysis can be applied to the banker. We illustrate numerically the theoretical solutions for banker and investor having both CRRA utilities.[19] In what follows, we examine three main cases: The first one corresponds to an investor who has no direct access to the financial derivatives market. Due to the suboptimality of his standard buy-and-hold portfolio, he may be ready to bear (theoretically) an additional cost to can include options in his portfolio. The second case deals with the banker's compensating variation due to riskier financial position in the presence of the investor's guarantee and/or the bad fit of his resulting constrained portfolio to his own risk aversion. Finally, we take the standard OBPI strategy as benchmark to both evaluate the investor's and banker's compensating variations.

## 4.1 Compensating variation of buy-and-hold strategy w.r.t. optimal portfolio with derivatives

In this subsection, we consider an investor who can only use a buy-an-hold strategy if he has no access to the financial derivatives market. We determine the compensating variation with respect to the true optimal payoff with derivatives provided by the bank. The level of the compensating variation provides a measure of the monetary loss, due to this friction. Conversely, they allow to measure the interest of the investor to use financial intermediates to manage their portfolios, which provides values of the cost they may accept to benefit from such financial service.[20]

[19] The other cases (logarithm and CARA) can be illustrated as well. However, CRRA utilities generally fit better the true utility. Additionnally, as shown for the CRRA case, the numerical values of the compensating variations are sufficiently significant to illustrate them.

[20] We assume that the investor cannot get a portfolio payoff which involves derivatives covered by a dynamic strategy in continuous-time. Thus, if he wants to include options in

In what follows, we assume that the risky asset price is a geometric Browbian motion (GBM) given by:

$$dS_t = S_t(\mu dt + \sigma dW_t). \tag{4}$$

Our numerical base case corresponds to the following financial parameters:

$$r = 3\%; \mu = 7\%; \sigma = 20\%; B_0 = 1; S_0 = 100; T = 5; V_0 = 1000; p = 1, \tag{5}$$

where $p$ denotes the guaranteed proportion.

#### 4.1.1 The optimal buy-and-hold strategy

The investor $i$ maximizes his expected utility:

$$Max_{w_{i,S}} \mathbb{E}\left[U\left[V_{i,T}\right]\right],$$

where $V_{i,T}$ denotes the portfolio value at maturity $T$ and $w_{i,S}$ corresponds to the proportion of wealth invested on the risky asset $S$. The buy-an-hold condition consists in fixing shares during the management period. Therefore, we have:

$$V_{i,T} = V_{i,0} \times \left(e^{rT} + w_{i,S}(e^{X_T} - e^{rT})\right)$$

The first-order condition implies:

$$\mathbb{E}\left[U'(V_{i,T})(e^{X_T} - e^{rT})\right] = 0,$$

which is equivalent to:

$$\int U'\left[V_{i,0} \times \left(e^{rT} + w^*_{i,S}(e^x - e^{rT})\right)\right](e^x - e^{rT}) f_X(x)dx = 0, \tag{6}$$

where $f_X$ denotes the pdf of the random variable $X = (\mu - 1/2\sigma^2)T + \sigma W_T$.

We can also introduce a specific guarantee constraint and solve the previous maximization problem under this additional condition. If the investor has not access to derivatives, then his utility level is smaller than when the bank provides his true optimal portfolio with derivatives. In what follows, we illustrate the compensating variation corresponding to this particular case.

#### 4.1.2 The compensating variation for the buy-and-hold strategy

For the CRRA case with guarantee, Bertrand *et al.* (2001) prove that the optimal portfolio payoff is given by:

$$V_{i,T} = p_i V_{i,0} + Max\left[\alpha_i V_{i,0} S_T^{m_i} - p_i V_{i,0}, 0\right])$$

his portfolio, he must either buy them on the financial market (with a given additional cost), either buy part of a structured fund managed by a financial institution (assumed here to be a bank for simplicity).

In this framework, Equation (3) is equivalent to:

$$\mathbb{E}[U_{\gamma_i}(p_iV_{i,0}+Max\left[\alpha_iV_{i,0}S_T^{m_i}-p_iV_{i,0},0\right])]=\mathbb{E}[U_{\gamma_i}(\tilde{V}_{i,0}\times\left(e^{rT}+w_{i,S}^*(e^x-e^{rT})\right))], \tag{7}$$

where $\alpha_i$ is determined from the budget equation:

$$\mathbb{E}_{\mathbb{Q}}\left(p_iV_{i,0}+Max\left[\alpha_iV_{i,0}S_T^{m_i}-p_iV_{i,0},0\right]\right)=V_0e^{rT}$$

**Proposition 4** *Under the CRRA assumption $U_{\gamma_i}(v)=\frac{v^{1-\gamma_i}}{1-\gamma_i}$, we deduce that the CV, equal to the ratio $\tilde{V}_{i,0}/V_{i,0}$, is given by:*

$$\frac{\tilde{V}_{i,0}}{V_{i,0}}=\left(\frac{\mathbb{E}_{\mathbb{P}}\left[\left(p_i+Max\left[\alpha_iS_T^m-p_i,0\right]\right)^{(1-\gamma)}\right]}{\mathbb{E}_{\mathbb{P}}[(e^{rT}+w_{i,S}^*\left[e^x-e^{rT}\right])^{(1-\gamma)}]}\right)^{\left(\frac{1}{1-\gamma_i}\right)}.$$

The numerical values of the CV $\frac{\tilde{V}_0}{V_0}$ are displayed in Table 5. We consider three values for both the drift $\mu$ and the volatility $\sigma$. Five levels of relative risk aversion (RRA) are introduced: $\gamma=0.5$ and 2 ("aggressive investors"); $\gamma=5$ (moderate investor); $\gamma=7$ and $\gamma=10$ (more conservative investors). The parameter $w_{i,S}$ denotes the optimal weights invested on the risky asset for the buy-and-hold strategy without insurance constraint. The parameter $w_{i,S}^c$ denotes the optimal weights invested on the risky asset for the buy-and-hold strategy with insurance constraint (the upper bound to guarantee the capital at maturity corresponds to $1-p_ie^{-rT}$ with the insured proportion $p_i$ equal to 1). The compensating variation (CV) is expressed in term of percentage of the true initial wealth invested $V_{i,0}$ on the financial market.

Table 5: Compensating Variations for the CRRA case, w.r.t. the standard buy-and-hold portfolio

| RRA $\gamma$ | $\mu=5\%$ and $\sigma=20\%$ | | | $\mu=7\%$ and $\sigma=20\%$ | | | $\mu=10\%$ and $\sigma=20\%$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | $w_S$ | $w_S^c$ | CV | $w_S$ | $w_S^c$ | CV | $w_S$ | $w_S^c$ | CV |
| 0.5 | 100% | 14% | 6.45% | 100% | 14% | 6.42% | 100% | 14% | 15.8% |
| 2 | 55% | 14% | 2.37% | 80% | 14% | 2.37% | 100% | 14% | 6.18% |
| 5 | 0% | 0% | 2% | 30% | 14% | 1.01% | 45% | 14% | 2.24% |
| 7 | 0% | 0% | 1.56% | 10% | 10% | 1.16% | 33% | 14% | 1.39% |
| 10 | 0% | 0% | 0.59% | 0% | 0% | 1.34% | 0% | 0% | 2.88% |
| RRA $\gamma$ | $\mu=7\%$ and $\sigma=15\%$ | | | $\mu=7\%$ and $\sigma=20\%$ | | | $\mu=7\%$ and $\sigma=25\%$ | | |
| | $w_S$ | $w_S^c$ | CV | $w_S$ | $w_S^c$ | CV | $w_S$ | $w_S^c$ | CV |
| 0.5 | 100% | 14% | 15.71% | 100% | 14% | 6.41% | 100% | 14% | 5.07% |
| 2 | 100% | 14% | 6.18% | 80% | 14% | 2.37% | 50% | 14% | 2.43% |
| 5 | 45% | 14% | 2.24% | 30% | 14% | 1.01% | 0% | 0% | 2% |
| 7 | 34% | 14% | 1.38% | 10% | 10% | 1.16% | 0% | 0% | 1.73% |
| 10 | 0% | 0% | 2.92% | 0% | 0% | 1.34% | 0% | 0% | 1.55% |

From Table (5), we note that the compensating variation can be relatively high (up to 15.8%). It is not a monotone function with respect to financial parameters $\mu$ and $\sigma$. However, for weak relative risk aversion ($\gamma_i = 0.5$ or 2), the values of compensating variation are higher than 2.37%. This is due to the fact that these investors are rather agressive once the guarantee is fixed. Therefore, they suffer from not having a strictly convex portfolio profile when using a buy-and-hold strategy. For conservative investors ($\gamma_i = 7$ or 10), the utility loss is less severe since they do not try to make the best benefit from the performance of the risky asset. For a moderate investor ($\gamma_i = 5$), the compensating variation is about 2%.

## 4.2 Compensating variations with constraints on the banker

In what follows, we assume that the banker provides exactly the optimal payoff with insurance constraint of the customer. Therefore, the banker may bear a utility loss for not having his own optimal portfolio profile. Indeed, his portfolio profile $h_b$ is determined from derivative market clearing conditions. Therefore, we have:

$$h_{b,T} = q_S S_T + q_B B_T - h_{i,T}, \tag{8}$$

where $q_S$ and $q_B$ are the shares respectively invested by the banker on the risky asset $S$ and the riskless asset $B$, on one hand to partially hedge the payoff $h_i$ sold to the investor and, on the other hand, to partially optimize the expected utility of his financial position $h_b$.

The parameters $q_S$ and $q_B$ are linked also to the budget constraint of the banker: $V_{b,0} = q_S S_0 + q_B B_0 - V_{i,0}$ and his regulatory risk constraint:

$$\mathbb{P}\left[V_{b,T} - V_{b,0} + VaR\left(\varepsilon\right) \leq 0\right] \leq \varepsilon, \tag{9}$$

where $VaR\left(\varepsilon\right)$ denotes the Value-at-Risk of the financial position at the given probability level $\varepsilon$.

Additional criteria can be introduced to determine the shares $q_S$ and $q_B$:

Criterion 1: The banker tries to minimize the ratio of the reserve amount upon the initial investment necessary to prevent losses at the given probability level $\varepsilon$.

Criterion 2: ("delta-neutrality") The portfolio manager (the banker) can try to hedge the investor's portfolio profile by setting:

$$q_S = \frac{\partial h_{i,0}}{\partial S_0}. \tag{10}$$

It means that the banker searches for a riskless portfolio but can only hedge the investor's risky position by a static position. Therefore, this leads only to a partial hedging with a residual risk, according to risky asset fluctuations.

### 4.2.1 Compensating variations with VaR constraints on the banker

The VaR condition can be detailed as follows:

$$\mathbb{P}\left[q_S S_T + q_B B_T - h_{i,T} - V_{b,0} + VaR\left(\varepsilon\right) \leq 0\right] \leq \varepsilon \tag{11}$$

which is equivalent to: (recall that $q_S S_0 + q_B B_0 - V_{i,0} = V_{b,0}$, thus $q_B = (V_{i,0} + V_{b,0} - q_S S_0)/B_0$).

$$\mathbb{P}\left[q_S\left(S_T - S_0 e^{rT}\right) - h_{i,T} + e^{rT} V_{i,0} + \left(e^{rT} + \delta - 1\right) V_{b,0} \leq 0\right] \leq \varepsilon,$$

with $VaR\left(\varepsilon\right)/V_{b,0} = \delta$.

Set:

$$c = -q_S S_0 e^{rT} + \left(e^{rT} - p_i\right) V_{i,0} + \left(e^{rT} + \delta - 1\right) V_{b,0}.$$

Then, the VaR condition is equivalent to:

$$\mathbb{P}\left[q_S S_T + V_{i,0} Max(\alpha_i S_T^{m_i} - p_i, 0) + c \leq 0\right] \leq \varepsilon. \tag{12}$$

The previous probability (12) is given by:

$$\mathbb{E}\left[\mathbb{I}_{q_S s + V_{i,0} Max(\alpha_i s^{m_i} - p_i, 0) + c \leq 0}\right].$$

In this framework, the characterization of the banker's compensating variation $CV_b$ is determined from the equality:

$$\mathbb{E}[U_b(h^*_{b,T}); V_{b,0})] = \mathbb{E}[U_b(h_{b,T}); \tilde{V}_{b,0}], \tag{13}$$

where $h^*_{b,T}$ denotes the true optimal payoff for the initial invested amount $V_{b,0}$.

Assume that both the banker and the investor have CRRA utilities with respective RRA $\gamma_b$ and $\gamma_i$. We consider both three values for the drift $\mu$ ($\mu = 4\%; \mu = 7\%; \mu = 15\%$) and for the volatility $\sigma$ ($\sigma = 10\%; \sigma = 20\%; \sigma = 30\%$). Five levels of the investor's relative risk aversion (RRA) are introduced: $\gamma_i = 0.5$ and 2 ("aggressive investors"); $\gamma_i = 5$ (moderate investor); $\gamma_i = 7$ and 10 (more conservative investors). For the banker, we assume the following relative risk aversions: $\gamma_b = 0.1$, 0.5, 2, 5 and 10. We choose the standard probability level: $\varepsilon = 1\%$. For the ratio , we set $\delta = 5\%$.[21]

In Appendix 4, figures about both investor's and banker's payoffs are displayed to illustrate the impact of expected risky asset return $\mu$ and volatility $\sigma$ on the portfolio profiles. We note for example that for high values of $\mu$ or weak values of $\sigma$ (thus high values of Sharpe ratio, which implies rather good performance of the financial market), the constrained banker's payoff can be, on one hand negative for risky asset values far from the spot value $S_0$, and, on the other hand, decreasing when the risky asset price rises significantly (see Figures A.4.3 and A.4.4). On the contrary, for small risky asset prices or high volatility values (thus small values of Sharpe ratio, which implies weak performance of the

[21] If $Min_{q_S, q_B} VaR(\varepsilon)/V_{b,0} \geq \delta$, we choose the pair $(q_S, q_B)$ that minimizes $VaR(\varepsilon)/V_{b,0}$.

financial market), the constrained banker's payoff is most of the time increasing for risky asset values far from the spot value $S_0$ (see Figures A.4.2 and A.4.5).

Due to Relation (8), the expected utility of the banker when accepting to sell the investor's profile is given by:

$$\frac{1}{1-\gamma_b}\mathbb{E}\left[\left(q_S S_T + q_B B_T - \left(p_i + Max\left[\alpha_i S_T^{m_i} - p_i, 0\right]\right) V_{i,0}\right)^{1-\gamma_b}\right].$$

His expected utility for his best strategy is given by:

$$\frac{1}{1-\gamma_b}\mathbb{E}\left[\left(\alpha_b \times S_T^{m_b}\right)^{1-\gamma_b}\right].$$

To compensate (theoretically) the non optimality of first portfolio, the banker must invest a higher initial amount $\tilde{V}_{b,0}$. Therefore, we deduce:

**Proposition 5** *The CV equal to the ratio $\tilde{V}_{b,0}/V_{b,0}$ is given by:*

$$\frac{\tilde{V}_{b,0}}{V_{b,0}} = \left(\frac{\mathbb{E}\left[\left(\alpha_b \times S_T^{m_b}\right)^{1-\gamma_b}\right]}{\mathbb{E}\left[\left(\frac{q_S S_T + q_B B_T - \left(p_i + Max\left[\alpha_i S_T^{m_i} - p_i, 0\right]\right) V_{i,0}}{\tilde{V}_{b,0}}\right)^{1-\gamma_b}\right]}\right)^{\left(\frac{1}{1-\gamma_b}\right)}.$$

The numerical values of the CV $\frac{\tilde{V}_{b,0}}{V_{b,0}}$ are displayed in Table (6), which provides the CV values for the VaR criterion. We set $V_{0,b} = 1000$.[22] For the banker, we assume the following relative risk aversions: $\gamma_b = 0.1$, 0.5, 2, 5 and 10. We can see that for high Sharpe type ratio due to high return ($\frac{\mu - r}{\sigma^2} = 3$ for the case $\mu = 15\%$ and $\sigma = 20\%$), the (theoretical) CV is very high (except for the less risky position corresponding to $\gamma_i = 10$ and $\gamma_b = 10$). This is mainly due to the high risk of the constrained banker's payoff, due to the convexity of the investors' payoff and/or his additional guarantee demand. Indeed, in this case, the static VaR hedge is inefficient (recall that $T = 5$ years) and the banker's risk is too high, which implies very high CV to theoretically compensate this risk level. For high Sharpe type ratio due to small volatility ($\frac{\mu - r}{\sigma^2} = 4$ for the case $\mu = 7\%$ and $\sigma = 10\%$), the CV lies between 0.2% and 10%. For $\gamma_b = 10$, the CV value is about 2%. These CV values are much smaller than previous ones, since the volatility risk is small. For more standard Sharpe type ratio ($\frac{\mu - r}{\sigma^2} = 1$ for the case $\mu = 7\%$ and $\sigma = 20\%$), the (theoretical) CV can be very high for weak RRA $\gamma_b$ of the banker ($\gamma_b = 0.1; 0.5$) and/or for weak RRA $\gamma_i$ of the investor. However, if we assume that the banker does not search for high returns from his investment $V_{b,0}$ but rather for total returns that are almost riskless, then his RRA $\gamma_b$ must be relatively high, for instance $\gamma_b = 10$. In that case, his CV lies

[22] It corresponds to the amount invested by the investor and allows to partially hedge the risk due to the investor's profile. Note that other banker's amount (for instance twice) do not change substantially the main conclusion of this study.

between 4.7% and 7.5%, except for the riskier case ($\gamma_i = 0.5$). For relatively small Sharpe type ratio ($\frac{\mu - r}{\sigma^2} = \frac{4}{9}$, for the case $\mu = 7\%$ and $\sigma = 30\%$), the CV is still high (about 23%) since the volatility is also very significant ($\sigma = 30\%$). Finally, for small Sharpe type ratio ($\frac{\mu - r}{\sigma^2} = \frac{1}{4}$, for the case $\mu = 4\%$ and $\sigma = 20\%$), the CV value lies mainly between 4% and 9%. This is due in particular to the bad performance of the financial market for the investor.

Table 6: CV CRRA (banker) VaR constraint

| RRA | $\mu = 4\%$ and $\sigma = 20\%$ | | | $\mu = 7\%$ and $\sigma = 20\%$ | | | $\mu = 15\%$ and $\sigma = 20\%$ | | |
|---|---|---|---|---|---|---|---|---|---|
| $\gamma_i/\gamma_b$ | 2 | 5 | 10 | 2 | 5 | 10 | 2 | 5 | 10 |
| 0.5 | 4.42% | 4.85% | 6% | 47% | 32% | 20% | ×65 | ×20 | ×54 |
| 2 | 4.88% | 6.98% | 9.83% | 6.5% | 4.6% | 4.7% | ×28 | ×10 | ×20 |
| 5 | 4.47% | 6.32% | 8.97% | 5.2% | 5.3% | 5.4% | ×1.36 | 13% | 6.5% |
| 10 | 4.33% | 6.09% | 8.54% | 4.8% | 4.9% | 7.5% | ×1.26 | 7.5% | 5% |
| RRA | $\mu = 7\%$ and $\sigma = 10\%$ | | | $\mu = 7\%$ and $\sigma = 20\%$ | | | $\mu = 7\%$ and $\sigma = 30\%$ | | |
| $\gamma_i/\gamma_b$ | 2 | 5 | 10 | 0.1 | 0.5 | | 2 | 5 | 10 |
| 0.5 | 7.72% | 7.75% | 8.2% | 83% | 85% | | 26% | 28% | 34% |
| 2 | 10.6% | 9.5% | 9.1% | 69% | 19.7% | | 23% | 24.5% | 27% |
| 5 | 26% | 13% | 9.8% | 64% | 16% | | 21.8% | 23.7% | 26.7% |
| 10 | 8.8% | 0.22% | 1.84% | 64% | 16% | | 21.3% | 23% | 25.8% |

### 4.2.2 Compensating variations with risk-neutral hedge constraints on the banker

Assume the delta-neutrality characterized by the condition: $q_S = \frac{\partial V_{i,0}}{\partial S_0}$. Here, since we have $V_{i,0} = \left(p_i e^{-rT} V_{i,0} + C_i\right)$ where denotes the initial value of the power call $Max\left[\alpha_i V_{i,0} S_T^{m_i} - p_i V_{i,0}, 0\right]$.

**Lemma 6** *Assume that $S$ is the geometric Brownian motion given in (4). Then the share $q_S = \frac{\partial V_{i,0}}{\partial S_0}$ is given by:*

$$q_S = \alpha_i V_{i,0} \left(m_i S_0^{m_i - 1} exp\left[1/2\ \sigma^2 m_i (m_i - 1) T\right]\right) N\left[d_1\left(\widehat{S}_0, \widehat{K}, \widehat{\sigma}, \widehat{r}\right)\right], \quad (14)$$

*where:*

$$\begin{aligned} \widehat{S}_0 &= S_0^{m_i} exp\left[1/2\ \sigma^2 m_i (m_i - 1) T\right], \widehat{K} = p_i/\alpha_i, \\ \widehat{\sigma} &= m_i \sigma, \text{ and } \widehat{r} = r. \end{aligned}$$

We illustrate now numerically this particular case for our numerical base case (5). For the numerical base case, we get the following investor and banker profiles: the banker and the investor have CRRA utilities with respective RRA $\gamma_b$ and $\gamma_i$. In Appendix (A.5), we illustrate both the investor's and banker's

portfolio profiles. We consider both three values for the drift $\mu$ ($\mu = 4\%; \mu = 7\%; \mu = 15\%$) and for the volatility $\sigma$ ($\sigma = 10\%; \sigma = 20\%; \sigma = 30\%$). Five levels of the investor's relative risk aversion (RRA) are introduced: $\gamma_i = 0.5$ and 2 ("aggressive investors"); $\gamma_i = 5$ (moderate investor); $\gamma_i = 7$ and 10 (more conservative investors). For the banker, we assume the following relative risk aversions: $\gamma_b = 0.1$, 0.5, 2, 5 and 10.

In Appendix 4, figures about both investor's and banker's payoffs are provided to show the influence of expected risky asset return $\mu$ and volatility $\sigma$ on the portfolio profiles. We note for example that the banker's payoff is most of the time positive except for weak RRA $\gamma_i$ of the investor and/or high risky expected return $\mu$. However, for high risky expected return $\mu$, it is still positive for small RRA $\gamma_i$ (for example, $\gamma_i = 10$ in Figure A.5.3). For small volatility ($\sigma = 10\%$) and small RRA $\gamma_i$ (for example, $\gamma_i = 0.5$ in Figure A.5.4), the banker's payoff is flat. Note also that most of the time, it decreasing from a given value of the spot price $S_0$.

We now examine the numerical values of the CV $\frac{\tilde{V}_{b,0}}{V_{b,0}}$. The numerical values of the CV $\frac{\tilde{V}_{b,0}}{V_{b,0}}$ are displayed in Table (7), which provides the CV values for the risk-neutral hedge criterion. We still set $V_{0,b} = 1000$. We can see that for high Sharpe type ratio due to high return ($\frac{\mu - r}{\sigma^2} = 3$ for the case $\mu = 15\%$ and $\sigma = 20\%$), the (theoretical) CV is very high (except for the less risky position corresponding to $\gamma_i = 10$ and $\gamma_b = 10$. In that case, the CV is equal to 5.4%). As for the VaR constraint, this is due to the high risk of the constrained banker's payoff, due to the convexity of the investors' payoff and/or his additional guarantee constraint.[23]For high Sharpe type ratio due to small volatility ($\frac{\mu - r}{\sigma^2} = 4$ for the case $\mu = 7\%$ and $\sigma = 10\%$), the CV lies between 6% and 100%. For the most convenient banker's RRA $\gamma_b = 10$, the CV value is about 12%. These CV values are smaller than previous ones when $\gamma_b = 10$, since the volatility risk is small. For more standard Sharpe type ratio ($\frac{\mu - r}{\sigma^2} = 1$ for the case $\mu = 7\%$ and $\sigma = 20\%$), the (theoretical) CV is can be very high for weak RRA $\gamma_b$ of the banker ($\gamma_b = 0.1; 0.5$) and/or for weak RRA $\gamma_i$ of the investor. When the banker's RRA $\gamma_b$ is relatively high (more convenient case), for instance $\gamma_b = 10$, his CV is about 4% or 5%, except for the riskier case ($\gamma_i = 0.5$). For relatively small Sharpe type ratio ($\frac{\mu - r}{\sigma^2} = \frac{4}{9}$, for the case $\mu = 7\%$ and $\sigma = 30\%$), the CV is still high (about 20%) since the volatility is also very significant ($\sigma = 30\%$). Finally, for small Sharpe type ratio ($\frac{\mu - r}{\sigma^2} = \frac{1}{4}$, for the case $\mu = 4\%$ and $\sigma = 20\%$), the CV value lies mainly between 3% and 4.5%. As for the Var constraint case, this is due for instance to the bad performance of the financial market for the investor.

[23]As in previous VaR case, the static risk-neutral hedge is rather inefficient in that case (recall that $T = 5$ years). Therefore, the banker's risk is too high, which implies very high CV to theoretically compensate this risk level.

Table 7: CV CRRA (banker) Risk-neutral hedge

| RRA | $\mu = 4\%$ and $\sigma = 20\%$ | | | $\mu = 7\%$ and $\sigma = 20\%$ | | | $\mu = 15\%$ and $\sigma = 20\%$ | | |
|---|---|---|---|---|---|---|---|---|---|
| $\gamma_i/\gamma_b$ | 2 | 5 | 10 | 2 | 5 | 10 | 2 | 5 | 10 |
| 0.5 | 4.37% | 4.35% | 4.6% | 47% | 36% | 42% | 77% | 93% | 96% |
| 2 | 3.7% | 3.6% | 3.54% | 8.44% | 5.9% | 5.2% | 75% | 93% | 96% |
| 5 | 3.5% | 3.33% | 3.32% | 8% | 5.5% | 4.7% | 46% | 20% | 13% |
| 10 | 3.4% | 3.23% | 3.21% | 7.78% | 5.1% | 4.3% | 45% | 11.6% | 5.4% |
| RRA | $\mu = 7\%$ and $\sigma = 10\%$ | | | $\mu = 7\%$ and $\sigma = 20\%$ | | | $\mu = 7\%$ and $\sigma = 30\%$ | | |
| $\gamma_i/\gamma_b$ | 2 | 5 | 10 | 0.1 | 0.5 | | 2 | 5 | 10 |
| 0.5 | 68% | 86% | 91% | 91% | 87% | | 2.55% | 2.51% | 2.4% |
| 2 | 69% | 87% | 92.8% | 57% | 20.5% | | 20% | 20% | 19.8% |
| 5 | 78% | 99% | 5.5% | 69.2% | 21.6% | | 20.1% | 20% | 19.9% |
| 10 | 84% | 6% | 12% | 72.4% | 21.15% | | 19.9% | 19.9% | 19.8% |

Note also that if we compute other CV numerical values, we get in particular the following values for the small volatility case ($\mu = 7\%$ and $\sigma = 10\%$):

$$CV\left[\gamma i = 0.5, \gamma_b = 0.1\right] \simeq 0\% \text{ and } CV\left[\gamma i = 0.5, \gamma_b = 0.5\right] = 7.28\%$$
$$CV\left[\gamma i = 2, \gamma_b = 0.1\right] \simeq 0\% \text{ and } CV\left[\gamma i = 2, \gamma_b = 0.5\right] = 7.4\%$$
$$CV\left[\gamma i = 5, \gamma_b = 0.1\right] \simeq 0\% \text{ and } CV\left[\gamma i = 5, \gamma_b = 0.5\right] = 8.4\%$$
$$CV\left[\gamma i = 10, \gamma_b = 0.1\right] \simeq 0\% \text{ and } CV\left[\gamma i = 10, \gamma_b = 0.5\right] = 9\%$$

The small CV values for $\gamma_b = 0.1$ are due to the better fit of the risk-neutral hedge banker's portfolio to his true optimal portfolio for small volatility when he has little relative risk-aversion.

## 4.3 Compensating variations of both the investor and the banker with respect to the standard OBPI case

In this subsection, we consider an investor who cannot exactly get his optimal portfolio. As emphasized in de Palma and Prigent (2008, 2009), typically financial institutions provide a limited number of standardized portfolios which do not exactly match investor preferences. Mistreating the "demand side" can also yield to significant utility losses for the investor. In what follows, we illustrate this feature for investors who can only buy the standard OBPI strategy. This financial product with guarantee is the first one (see Leland and Rubinstein, 1976), and serves as fundamental example to construct other guarantee funds.

We determine the compensating variation with respect to the true optimal payoff. The levels of compensating variation provide a measure of the monetary loss, due to of this type of friction. We also examine the CV for the banker in this framework.

The OBPI method consists basically in purchasing an amount $q_i.K_i$ invested on the money market account, and $q_i$ shares of European call options written

on asset $S$ with maturity $T$ and exercise price $K_i$. The value $V_{i,t}^{OBPI}$ of this portfolio at any time $t$ in the period $[0,T]$ is:

$$V_{i,t}^{OBPI} = q_i \left( K_i e^{-r(T-t)} + C(t, K_i) \right), \tag{15}$$

where $C(t,x)$ denotes the no-arbitrage value of a European call option with strike $x$, calculated under a given risk-neutral probability $Q$ (if coefficient functions $\mu$, $a$ and $b$ are constant, $C(t,x)$ is the usual Black-Scholes value of the European call). Note that, for all dates $t$ before $T$, the portfolio value $V_{i,t}^{OBPI}$ is always above the deterministic level $q_i K_i e^{-r(T-t)}$. The investor is still willing to recover a percentage $p$ of his initial investment $V_{i,0}$. Then, the portfolio manager has to choose the two appropriate parameters, $q_i$ and $K_i$.

- First, since the insured amount is equal to $q_i.K_i$, it is required that $K_i$ satisfies the relation:[24]

$$p_i V_{i,0} = p_i q_i (K_i.e^{-rT} + C(0, K_i)) = q_i K_i, \tag{16}$$

which implies that:

$$\frac{C(0, K_i)}{K_i} = \frac{1 - p_i e^{-rT}}{p_i}. \tag{17}$$

Therefore, the strike $K_i$ is an increasing function $K_i\,(p_i)$ of percentage $p_i$.

- Second, the number of shares $q_i$ is given by:

$$q_i = \frac{V_{i,0}}{K_i e^{-rT} + C(0, K_i)}. \tag{18}$$

Thus, for any investment value $V_{i,0}$, number of shares $q_i$ is a decreasing function of percentage $p_i$.

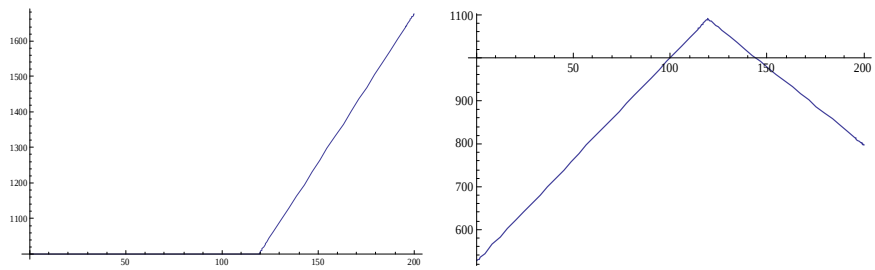
For our numerical base case (5), we get: $K_i = 119.3$ and $q_i = 8.38$.

Next figure illustrates both the investor's and banker's portfolio profiles when the OBPI portfolio is provided to the investor. For the banker, we consider the risk-neutral hedge portfolio with initial investment $V_{b,0} = 1000$.

As it can be seen, the investor's guarantee ($V_{i,0} \geq V_{i,T}$) implies that the banker's constrained portfolio may suffer from significant losses for bearish market (for example if $S_T \leq 0.8\ S_0$,then the loss is higher than 10%) or when the risky asset price $S_T$ grows highly ($S_T \geq 1.45\ S_0$).

We compute now the CV for both the investor and the banker, with same RRA values $\gamma_i$ and $\gamma_b$ as previously. Recall that the OBPI strategy is the optimal investor's portfolio if and only if $\gamma_i = 1/(\frac{\mu - r}{\sigma^2})$ (in that case, the power $m_i$ is equal to 1). Apart this case, the investor does not receive his true optimal

[24] This relation can also be adjusted to take account of the smile effect.




OBPI investor's portfolio profile

OBPI banker's portfolio profile

Figure 4: OBPI Profiles

portfolio and bears utility losses. The banker also has not his true optimal portfolio. Additionally, he may suffer from portfolio value losses. Consequently, we must examine both their compensating variations. They are displayed in next table. We note that in this framework, the CV is generally much higher for the banker than the investor. However, if we consider the standard case (risky asset return $\mu = 7\%$, which is the usual value on long term horizon and volatility $\sigma = 20\%$), then the investor's and banker's compensating variations are almost equal. Roughly speaking, it means that these two CV compensate each other, which implies that only small CV values can be required by the banker (if any).

Table 8: CV for the CRRA case (OBPI)

| RRA | $\sigma = 20\%$ | | | $\mu = 7\%$ | | |
|---|---|---|---|---|---|---|
| $\gamma_i/\sigma$ | $\mu = 4\%$ | $\mu = 7\%$ | $\mu = 15\%$ | $\sigma = 10\%$ | $\sigma = 20\%$ | $\sigma = 30\%$ |
| 0.5 | 5.36% | 15.2% | ×2.3 | ×2.1 | 15.2% | 19.4% |
| 2 | 7.52% | 6.5% | 21.6% | 20% | 6.5% | 19.7% |
| 5 | 11.7% | 9.2% | 9% | 10.3% | 9.2% | 24.5% |
| 10 | 14.1% | 12.7% | 11% | 10.1% | 12.7% | 28.6% |
| RRA | $\sigma = 20\%$ | | | $\mu = 7\%$ | | |
| $\gamma_b$ | $\mu = 4\%$ | $\mu = 7\%$ | $\mu = 15\%$ | $\sigma = 10\%$ | $\sigma = 20\%$ | $\sigma = 30\%$ |
| 0.5 | ×3.2 | 19.5% | ×6.2 | ×4.2 | 19.5% | ×3.4 |
| 2 | ×2.67 | 7.4% | ×5.9 | ×3.9 | 7.4% | ×2.4 |
| 5 | ×2.82 | 7.1% | ×6.3 | ×5.6 | 7.1% | ×2.7 |
| 10 | ×4.1 | 11.4% | ×6.5 | 5.2% | 11.4% | ×3.1 |

# 5 Conclusion

Using the notion of compensating variation, we provide a quantitative measure to evaluate the monetary loss of not having the best portfolio profile on one hand for the clients, on the other hand for the financial institutions. Therefore, we can evaluate the mispricing of structured products when they do not match exactly the investors preferences. Clearly the overevaluation usually observed depends on the risk aversion of the investor and on the specification of the financial contracts. We provide the numerical illustration of these theoretical compensations for banker and investor having both CRRA utilities, through three main cases: investor having no direct access to the financial derivatives market; banker's compensating variation due to riskier financial position induced by the investor's guarantee; and finally the standard OBPI strategy as benchmark to both measure the investor's and banker's compensating variations. For standard financial parameter values and rational relative risk aversions (it means in particular that the banker must have a significant risk aversion), the level of these compensating variations lies between 3% and 10% for the banker's compensating variation, when he bears the insurance constraint of the investor. However, if we compute both the banker's and the investor's compensating variations with respect to the benchmark OBPI strategy, the two values are very close. In this latter case, this balance between the compensating variations yields to small compensating variations at the equilibrium.

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**Appendix 1. Proof of Proposition 1.**

We price this financial asset in a Black and Scholes framework by taking the sum of the discounted expected value of the payoffs under the appropriate unique martingale measure. We have:

$$\begin{aligned}
V_0(S_0) &= V_0 e^{-rT}\mathbb{E}_{\mathbb{Q}}\left[1+\frac{(S_T-S_0)}{S_0}\mathbb{I}_{[S_0\leq S_T<2S_0]}+\mathbb{I}_{[S_T>2S_0]}\right], \\
&= V_0 e^{-rT}\mathbb{E}_{\mathbb{Q}}\left[1+\frac{(S_T-S_0)}{S_0}\mathbb{I}_{[S_T\geq S_0]}+\left(1-\frac{(S_T-S_0)}{S_0}\right)\mathbb{I}_{[S_T>2S_0]}\right], \\
&= V_0 e^{-rT}\mathbb{E}_{\mathbb{Q}}\left[1+\frac{(S_T-S_0)}{S_0}\mathbb{I}_{[S_T\geq S_0]}-\left(\frac{S_T-2S_0}{S_0}\right)\mathbb{I}_{[S_T>2S_0]}\right], \\
&= V_0 e^{-rT}+\frac{V_0}{S_0}\left[Call\left(T,S_0,S_0,r,\sigma\right)-Call\left(T,S_0,2S_0,r,\sigma\right)\right].
\end{aligned}$$

**Appendix 2. Proof of Proposition 2.**

We have:

$$V_0(S_0)=V_0e^{-rT}+\frac{0.75}{4T}\times$$

$$\sum\nolimits_{i=1}^{4T} e^{-r(T-\frac{i}{4})}e^{-r\frac{i}{4}}\mathbb{E}_{\mathbb{Q}}\left[\mathbb{I}_{[S_{i/4}\geq L.S_0]}\left\{(H-1)V_0+(S_{i/4}-HS_0)^{+}\frac{V_0}{S_0}\right\}\right],$$

thus:

$$V_0(S_0)=V_0e^{-rT}+\frac{0.75}{4T}\times$$

$$\frac{0.75}{4T}\sum\nolimits_{i=1}^{4T} e^{-r(T-\frac{i}{4})}e^{-r\frac{i}{4}}\left(V_0(H-1)\mathbb{E}_{\mathbb{Q}}\left[\mathbb{I}_{[S_{i/4}\geq L.S_0]}\right]+\mathbb{E}_{\mathbb{Q}}\left[(S_{i/4}-H.S_0)^{+}\frac{V_0}{S_0}\right]\right).$$

We need to compute the expectation of the indicator function:,

$$\begin{aligned}
\mathbb{E}_{\mathbb{Q}}\left[\mathbb{I}_{[S_{i/4}\geq L.S_0]}\right] &= \mathbb{Q}\left[S_{i/4}\geq L.S_0\right] \\
&= 1-\mathbb{Q}\left[S_{i/4}<L.S_0\right]
\end{aligned} \tag{19}$$

We adopt a simple continuous-time model where the stock index price dynamics is given by the following stochastic process :

$$dS_t=S_t[\mu dt+\sigma dW_t], \tag{20}$$

which implies:

$$S_t=S_0\exp[(\mu-\frac{1}{2}\sigma^2)t+\sigma W_t] \tag{21}$$

where $(W_t)_t$ is a standard Brownian motion with respect to a given filtration $(\mathcal{F}_t)_t$. This can be written as:

$$S_t=S_0\exp[(\mu-\frac{1}{2}\sigma^2)t+\sigma\sqrt{t}Y] \tag{22}$$

where $Y\sim\mathcal{N}(0,1)$.

Under $\mathbb{Q}$, we have:

$$S_t = S_0 \exp[(r - \frac{1}{2}\sigma^2)t + \sigma\sqrt{t}Y] \tag{23}$$

Thus, expression (19) becomes:

$$\mathbb{E}_{\mathbb{Q}}\left[\mathbb{I}_{[S_{i/4} \geq L.S_0]}\right] = 1 - N\left[d\left(i/4, L\right)\right]$$

where $d\left(i/4, L\right) = \frac{Ln(L) - \left(r - \frac{\sigma^2}{2}\right)\frac{i}{4}}{\sigma\sqrt{i/4}}$.

Finally, we obtain:

$$V_0(S_0) = V_0 e^{-rT} + \frac{0.75}{4T} \times$$

$$\sum\nolimits_{i=1}^{4T} e^{-r(T-\frac{i}{4})}\left[e^{-r\frac{i}{4}}(H-1)V_0\left[1 - N\left(d\left(i/4, L\right)\right)\right] + Call\left(i/4, S_0, HS_0, r, \sigma\right)\frac{V_0}{S_0}\right].$$

**Appendix 3. Proof of Lemma 6.**

The power call $Max\left[\alpha_i V_{i,0} S_T^{m_i} - p_i V_{i,0}, 0\right]$ is equal to

$$\alpha_i V_{i,0} Max\left[S_T^{m_i} - p_i/\alpha_i, 0\right].$$

Recall that we suppose:

$$S_t = S_0 exp\left[(\mu - 1/2\sigma^2)t + \sigma W_t\right].$$

Thus, we have:

$$S_T^{m_i} = S_0^{m_i} exp\left[m_i(\mu - 1/2\sigma^2)T + m_i\sigma W_T\right],$$

which is also equal to

$$S_0^{m_i} exp\left[1/2\ \sigma^2 m_i(m_i - 1)T\right].exp\left[(m_i\mu - 1/2m_i^2\sigma^2)T + m_i\sigma W_T\right].$$

Therefore, using the Black and Scholes formula for a standard call option with underlying $\widehat{S}_0$, strike $\widehat{K}$, volatility $\widehat{\sigma}$, interest rate $\widehat{r}$, the initial value of the power call is given by:

$$N\left[d_1\left(\widehat{S}_0, \widehat{K}, \widehat{\sigma}, \widehat{r}\right)\right] - N\left[d_2\left(\widehat{S}_0, \widehat{K}, \widehat{\sigma}, \widehat{r}\right)\right],$$

with:

$$\begin{aligned} d_1\left(\widehat{S}_0, \widehat{K}, \widehat{\sigma}, \widehat{r}\right) &= \frac{\ln\left(\frac{\widehat{S}_0}{\widehat{K}}\right) + \left(\widehat{r} + \frac{1}{2}\widehat{\sigma}^2\right)T}{\widehat{\sigma}\sqrt{T}}, \\ d_2\left(\widehat{S}_0, \widehat{K}, \widehat{\sigma}, \widehat{r}\right) &= d_1\left(\widehat{S}_0, \widehat{K}, \widehat{\sigma}, \widehat{r}\right) - \widehat{\sigma}\sqrt{T}, \end{aligned}$$

and:

$$\begin{aligned}
\widehat{S}_0 &= S_0^{m_i} exp\left[1/2\ \sigma^2 m_i(m_i-1)T\right], \\
\widehat{K} &= p_i/\alpha_i, \\
\widehat{\sigma} &= m_i\sigma, \\
\widehat{r} &= r.
\end{aligned}$$

Consequently, we have:

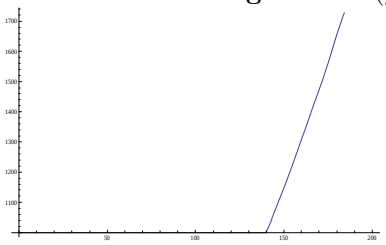
$$\frac{\partial V_{i,0}}{\partial S_0} = \alpha_i V_{i,0}\left(\frac{\partial \widehat{S}_0}{\partial S_0}.\frac{\partial V_{i,0}}{\partial \widehat{S}_0}\right).$$

Thus we have:

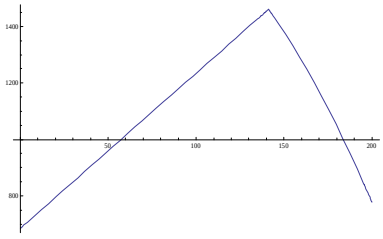
$$\frac{\partial V_{i,0}}{\partial S_0} = \alpha_i V_{i,0}\left(m_i S_0^{m_i-1} exp\left[1/2\ \sigma^2 m_i(m_i-1)T\right]\right) N\left[d_1\left(\widehat{S}_0, \widehat{K}, \widehat{\sigma}, \widehat{r}\right)\right].$$

**Appendix 4. (Graphical illustrations of the investor's and bankers's payoffs for VaR constraints)**

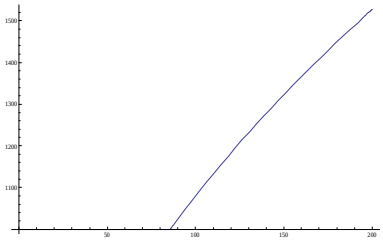
**Fig. A.4.1.** ($\mu = 7\%, \sigma = 20\%$)



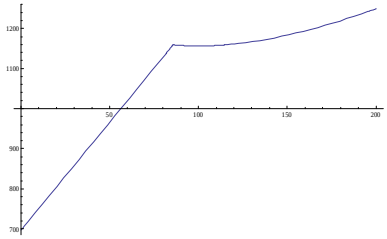
The investor's optimal profile ($\gamma_i = 0.5$)



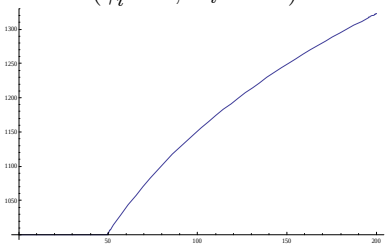
The banker's constrained profile $q_S = 5.5, q_B = 1450$)



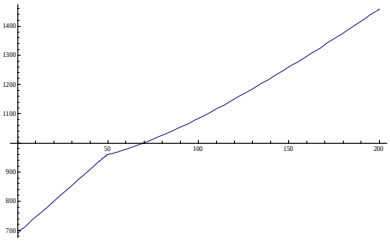
The investor's optimal profile ($\gamma_i = 2; m_i = 0.5$)



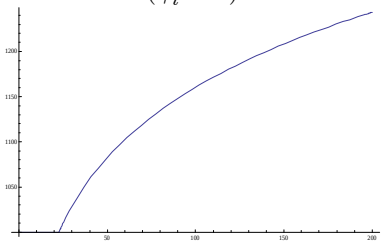
The banker's constrained profile ($q_S = 5.4, q_B = 1460$)



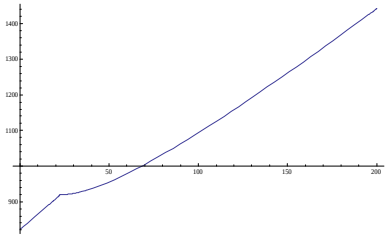
The investor's optimal profile ($\gamma_i = 5$)



The banker's constrained profile ($q_S = 5.45, q_B = 1455$)

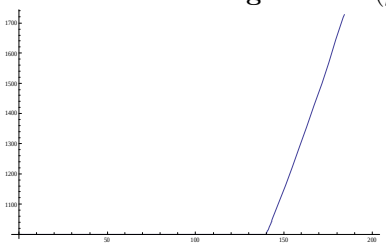


The investor's optimal profile ($\gamma_i = 10$)

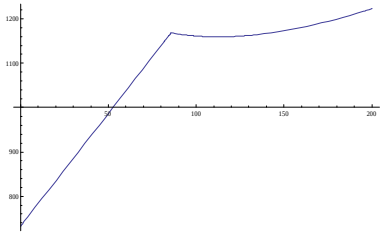


The banker's constrained profile ($q_S = 4.32, q_B = 1568$)

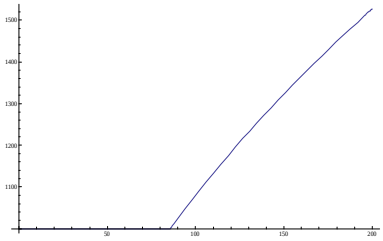
**Fig. A.4.2.** $(\mu = 4\%, \sigma = 20\%)$



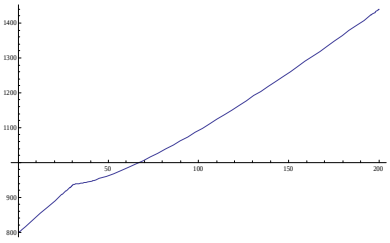
The investor's optimal profile
$(\gamma_i = 0.5)$



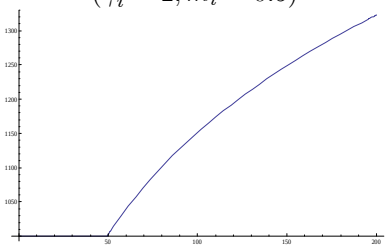
The banker's constrained profile
$(q_S = 5.09, q_B = 1491)$



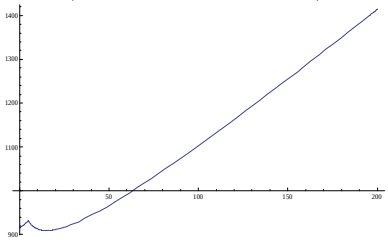
The investor's optimal profile
$(\gamma_i = 2; m_i = 0.5)$



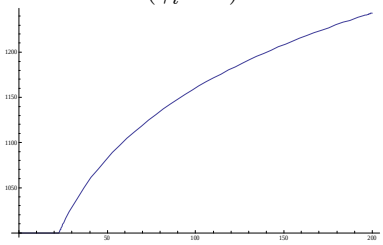
The banker's constrained profile
$(q_S = 4.52, q_B = 1548)$



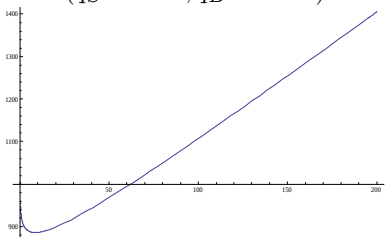
The investor's optimal profile
$(\gamma_i = 5)$



The banker's constrained profile
$(q_S = 3.17, q_B = 1683)$

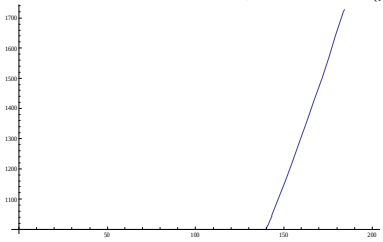


The investor's optimal profile
$(\gamma_i = 10)$

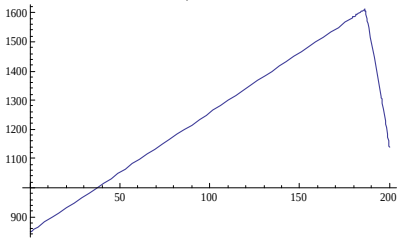


The banker's constrained profile
$(q_S = 3.20, q_B = 16.80)$

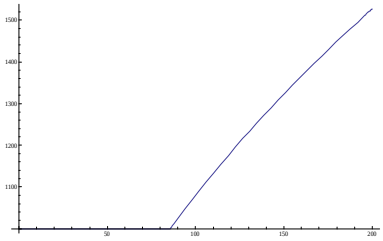
**Fig. A.4.3.** $(\mu = 15\%, \sigma = 20\%)$



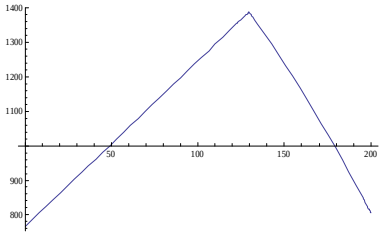
The investor's optimal profile
$(\gamma_i = 0.5)$



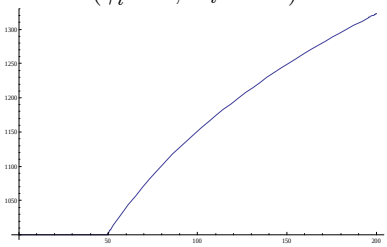

The banker's constrained profile
$(q_S = 4.1, q_B = 1590)$



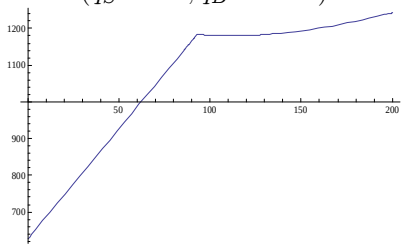
The investor's optimal profile
$(\gamma_i = 2; m_i = 0.5)$



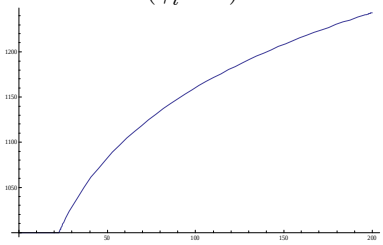

The banker's constrained profile
$(q_S = 4.8, q_B = 1520)$



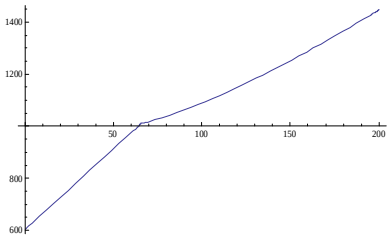
The investor's optimal profile
$(\gamma_i = 5)$




The banker's constrained profile
$(q_S = 6, q_B = 1400)$

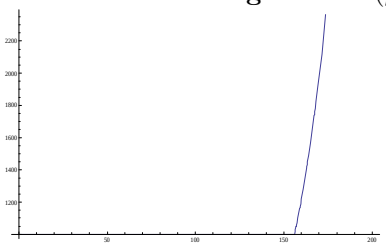


The investor's optimal profile
$(\gamma_i = 10)$

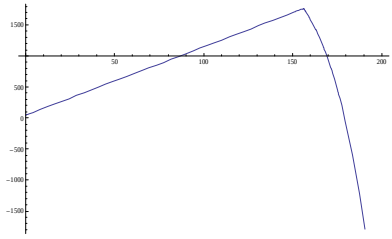



The banker's constrained profile
$(q_S = 6.2, q_B = 1480)$

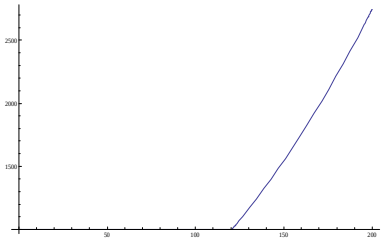
**Fig. A.4.4.** ($\mu = 7\%, \sigma = 10\%$)



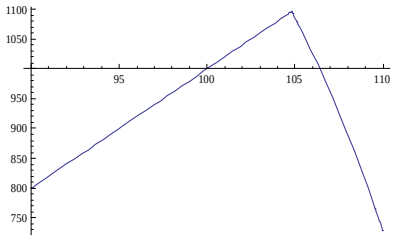
The investor's optimal profile ($\gamma_i = 0.5$)



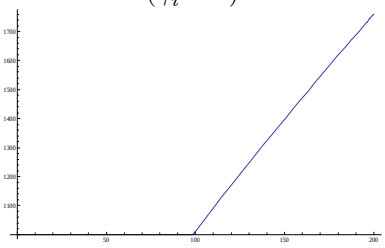
The banker's constrained profile ($q_S = 11, q_B = 900$)



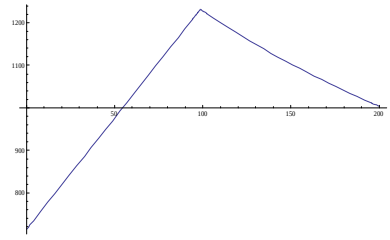
The investor's optimal profile ($\gamma_i = 2$)



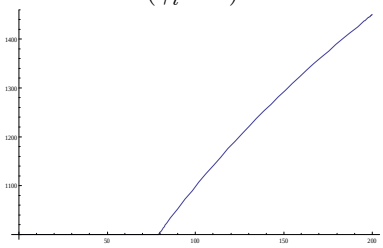
The banker's constrained profile ($q_S = 20, q_B = 0$)



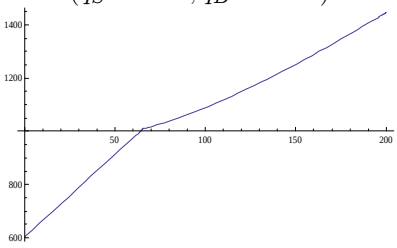
The investor's optimal profile ($\gamma_i = 5$)



The banker's constrained profile ($q_S = 5.25, q_B = 1475$)

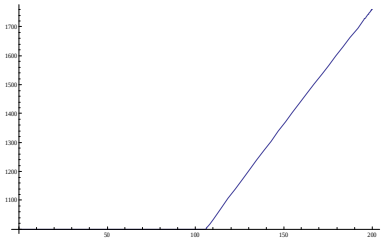


The investor's optimal profile ($\gamma_i = 10$)

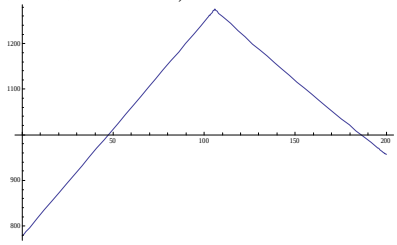


The banker's constrained profile ($q_S = 10.6, q_B = 940$)

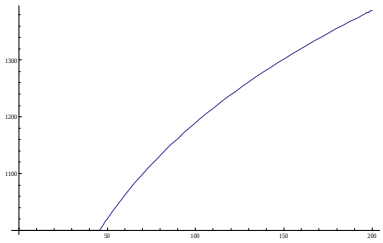
**Fig. A.4.5.** ($\mu = 7\%, \sigma = 30\%$)



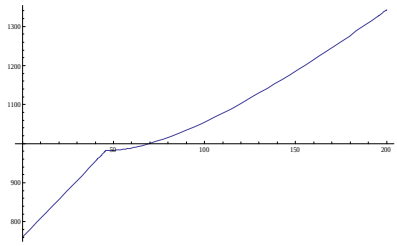
The investor's optimal profile ($\gamma_i = 0.5$)



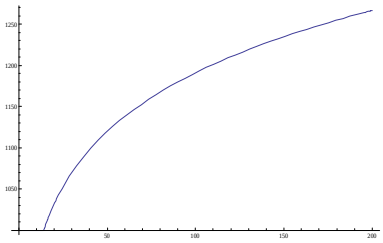
The banker's constrained profile ($q_S = 4.7, q_B = 1530$)



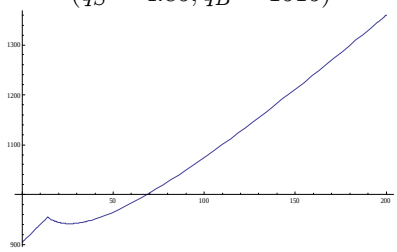
The investor's optimal profile ($\gamma_i = 2$)



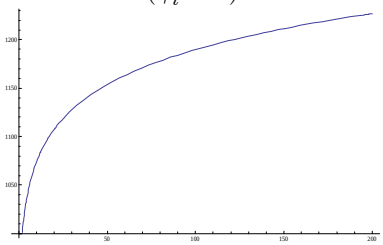
The banker's constrained profile ($q_S = 4.85, q_B = 1515$)



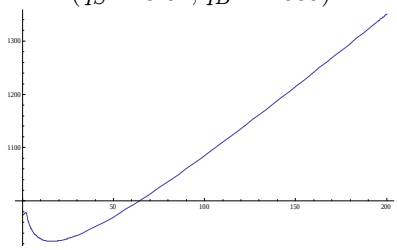
The investor's optimal profile ($\gamma_i = 5$)



The banker's constrained profile ($q_S = 3.61, q_B = 1639$)



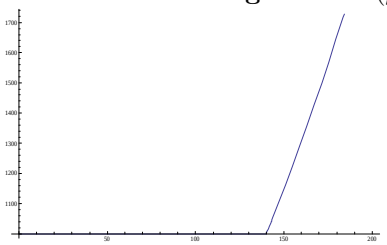
The investor's optimal profile ($\gamma_i = 10$)



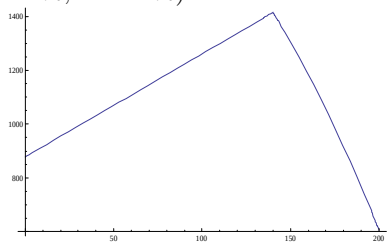
The banker's constrained profile ($q_S = 3.02, q_B = 1698$)

**Appendix 5. (Graphical illustrations of the investor's and bankers's payoffs for delta-neutrality constraints)**

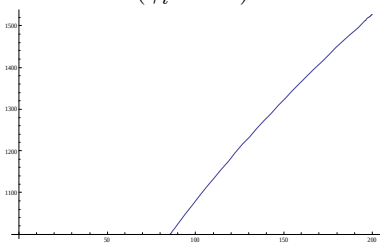
**Fig. A.5.1.** $(\mu = 7\%, \sigma = 20\%)$



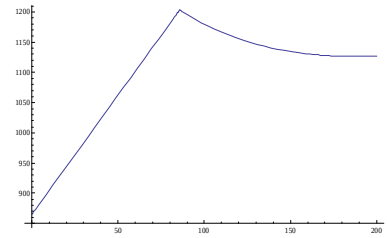
The investor's optimal profile
$(\gamma_i = 0.5)$



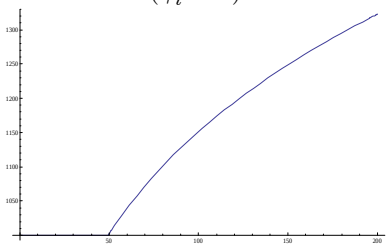
The banker's constrained profile
$(q_S = 3.84, q_B = 1616)$



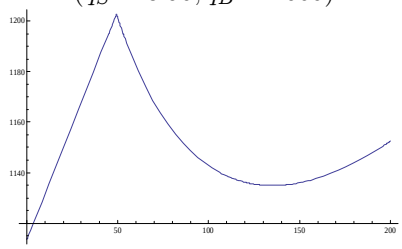
The investor's optimal profile
$(\gamma_i = 2)$



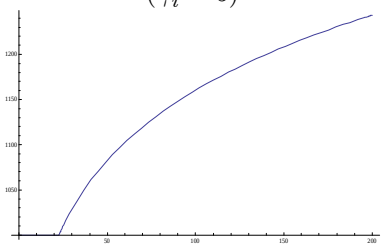
The banker's constrained profile
$(q_S = 3.95, q_B = 1605)$



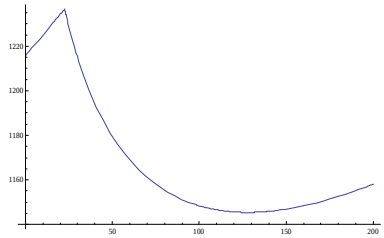
The investor's optimal profile
$(\gamma_i = 5)$



The banker's constrained profile
$(q_S = 1.81, q_B = 1819)$

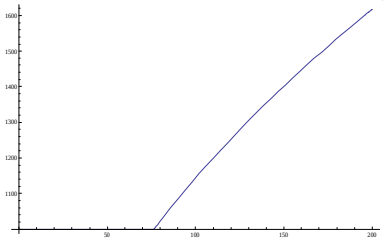


The investor's optimal profile
$(\gamma_i = 10)$

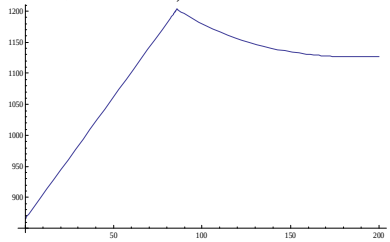


The banker's constrained profile
$(q_S = 0.93, q_B = 1907)$

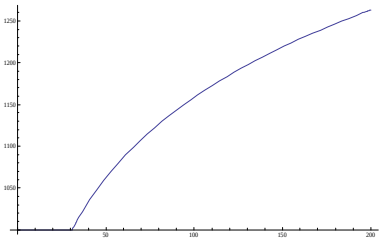
**Fig. A.5.2.** $(\mu = 4\%, \sigma = 20\%)$



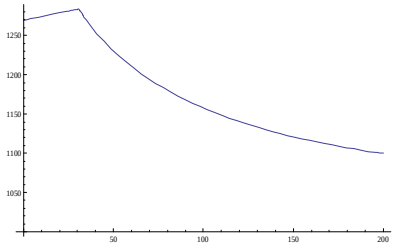
The investor's optimal profile
$(\gamma_i = 0.5)$



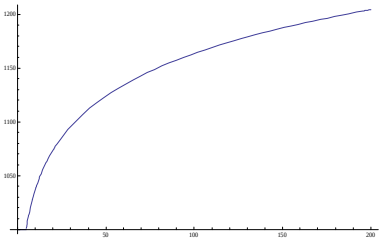
The banker's constrained profile
$(q_S = 3.95, q_B = 1605)$



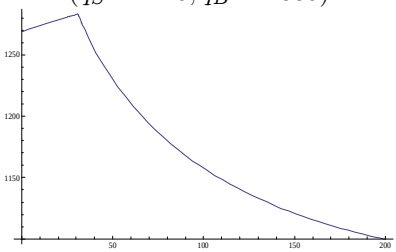
The investor's optimal profile
$(\gamma_i = 2)$



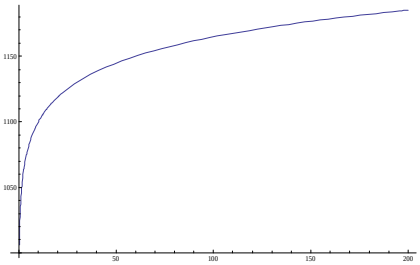
The banker's constrained profile
$(q_S = 1.15, q_B = 1885)$



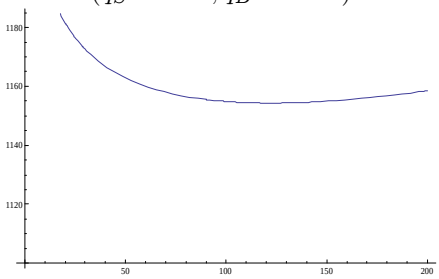
The investor's optimal profile
$(\gamma_i = 5)$



The banker's constrained profile
$(q_S = 0.47, q_B = 1953)$

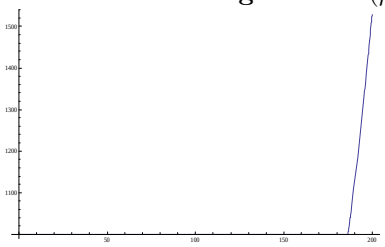


The investor's optimal profile
$(\gamma_i = 10)$

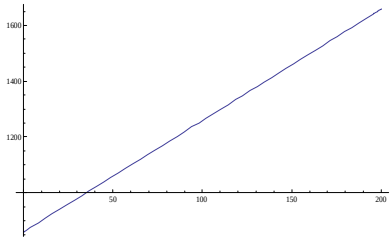


The banker's constrained profile
$(q_S = 0.24, q_B = 1976)$

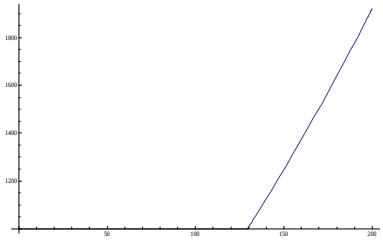
**Fig. A.5.3.** $(\mu = 15\%, \sigma = 20\%)$



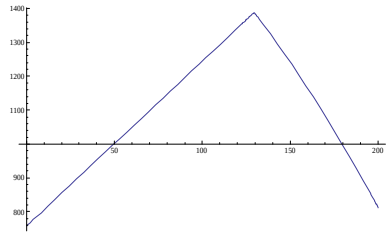

The investor's optimal profile
$(\gamma_i = 0.5)$



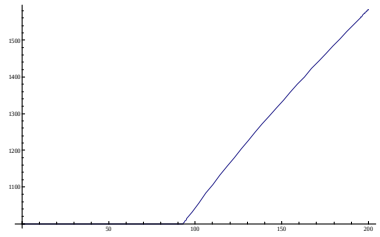

The banker's constrained profile
$(q_S = 4, q_B = 1600)$



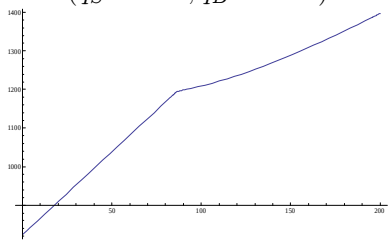

The investor's optimal profile
$(\gamma_i = 2)$



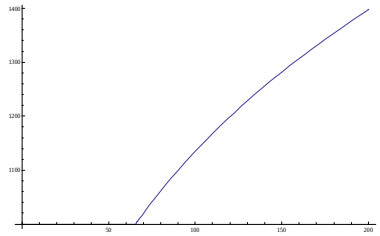

The banker's constrained profile
$(q_S = 4.88, q_B = 1512)$



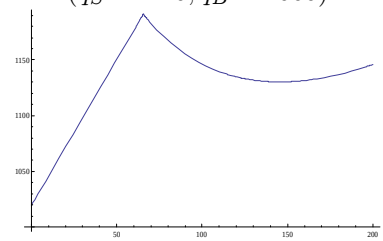

The investor's optimal profile
$(\gamma_i = 5; m_i = 3/5, \text{“}OBPI\text{”})$




The banker's constrained profile
$(q_S = 4.45, q_B = 1555)$

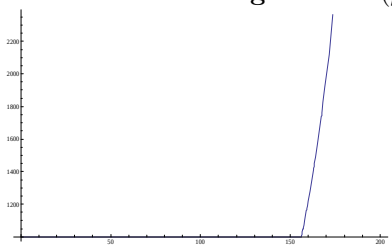



The investor's optimal profile
$(\gamma_i = 10)$

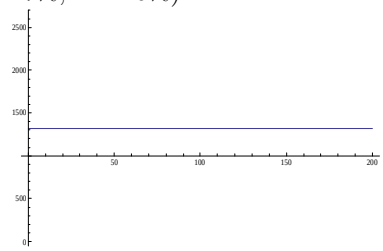



The banker's constrained profile
$(q_S = 2.62, q_B = 1738)$

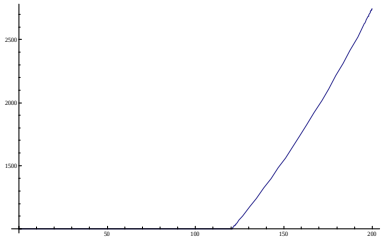
**Fig. A.5.4.** $(\mu = 7\%, \sigma = 10\%)$



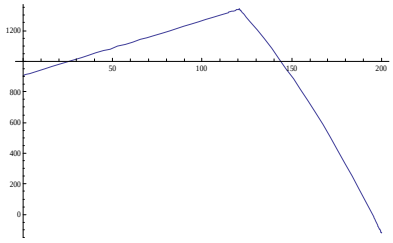

The investor's optimal profile
$(\gamma_i = 0.5)$



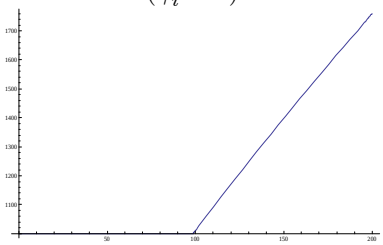

The banker's constrained profile
$q_S = 0, q_B = 2000)$



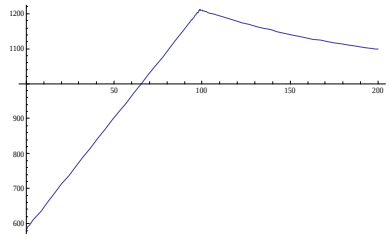

The investor's optimal profile
$(\gamma_i = 2)$



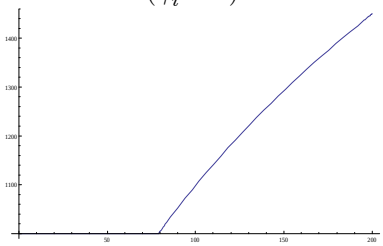

The banker's constrained profile
$(q_S = 3.6, q_B = 1640)$



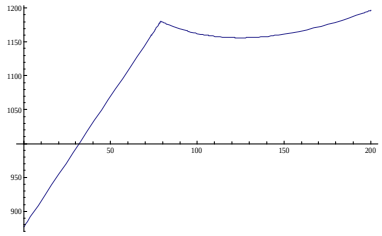

The investor's optimal profile
$(\gamma_i = 5)$




The banker's constrained profile
$(q_S = 6.38, q_B = 1362)$

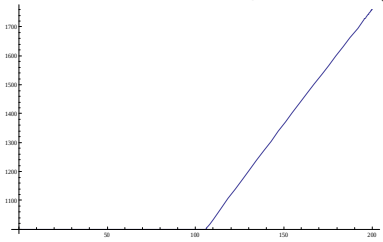



The investor's optimal profile
$(\gamma_i = 10)$

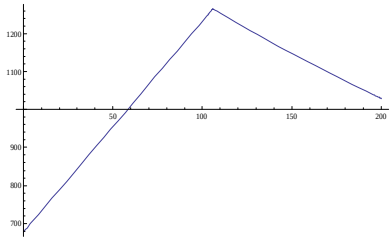



The banker's constrained profile
$(q_S = 3.85, q_B = 1615)$

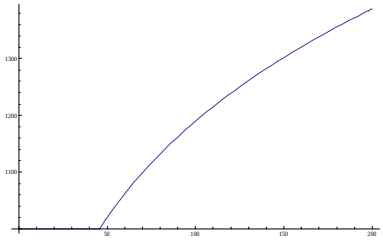
**Fig. A.5.5.** $(\mu = 7\%, \sigma = 30\%)$



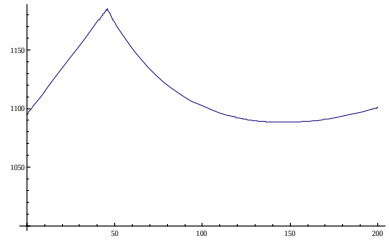
The investor's optimal profile
$(\gamma_i = 0.5)$



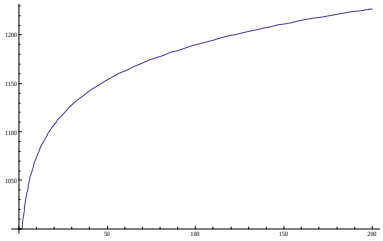
The banker's constrained profile
$q_S = 5.57, q_B = 1443)$



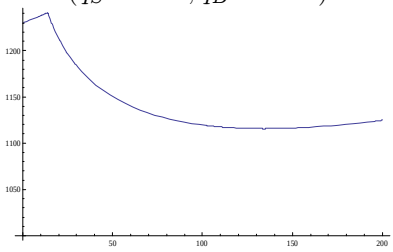
The investor's optimal profile
$(\gamma_i = 2)$



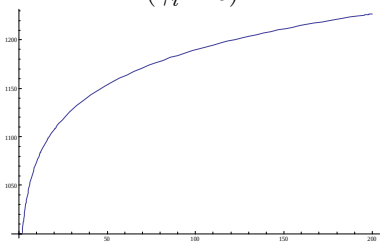
The banker's constrained profile
$(q_S = 1.97, q_B = 1803)$



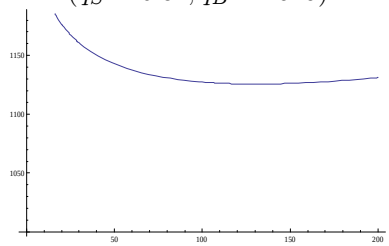
The investor's optimal profile
$(\gamma_i = 5)$



The banker's constrained profile
$(q_S = 0.81, q_B = 1919)$



The investor's optimal profile
$(\gamma_i = 10)$



The banker's constrained profile
$(q_S = 0.41, q_B = 1959)$