The impact of the recent financial crisis on Eurozone sovereign credit default swap spreads

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Abstract

We evaluate the effects of the recent financial crisis on an important class of debt instruments: sovereign issues of Eurozone borrowers. Challenges to the stability of the Euro from threats of default by several Eurozone countries have raised serious concerns and led to unprecedented policy responses. We propose to study these effects by evaluating the risk premia embedded in sovereign credit default swap (CDS) spreads during periods of financial turmoil. These instruments provide insurance to their buyers, payable in the event of default. Their spreads over riskless instruments and spreads within the Eurozone CDS universe provide direct indications of market participants' valuation of risk associated with the underlying sovereign debt.

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1 Introduction

One of the financial innovations highlighted as playing a key role in the recent financial crisis is the credit default swap (CDS): essentially, an insurance policy on the payments forthcoming from a debt instrument. Although these instruments were maligned in the US market as having largely contributed to the demise of insurer AIG, it should be clear that writing these instruments is no different than selling put options, and can lead to equally disastrous consequences should the underlying debt instruments become 'puttable' through a credit event.

On the positive side, the development of an active CDS market has been beneficial in providing rapid feedback on market participants' perception of the riskiness of debt instruments: in particular, sovereign debt issued by European borrowers. Unlike infrequently-revised credit ratings, which were also heavily maligned in the collapse of US mortgage-backed securities markets, quoted CDS spreads—like market yields—adjust quickly to market conditions and perceptions. For instance:

> "The attitude of the [European] commission also suggests a certain paranoia that European sovereign issuers are being treated unfairly by ratings agencies. The reverse is the case. French ten-year bonds trade on a yield more than a percentage point higher than their German equivalents, even though both share a AAA rating. An analysis by Exotix, a research group, found that peripheral European nations have ratings six notches higher than emerging economies with similar financial conditions."[1]

Given the importance of CDS quoted spreads as a measure of market participants' perceptions of default risk, we seek to exploit the information in the set of spreads on Eurozone sovereign issuers during 2009–2010. Unlike much of the prior literature, we make use of daily quotations rather than monthly quotations on these instruments. This provides us with not only a much larger sample size during this turbulent period, but also allows us to more accurately evaluate the contagion effects that may be present between different sovereign borrowers' spreads as sizable uncertainty over the fate of the most troubled economies washes over the market.

[1] 'The road to self-deception', *The Economist*, Nov, 12, 2011, p. 84.

Our empirical investigation begins with a set of unconditional statistics on the interrelations between Eurozone sovereign borrowers, followed by the development of a model of the dynamic interaction effects of CDS spreads across borrowers, conditioned on standard market evaluations of the probability of default. This allows us to produce static forecasts of CDS spreads' evolution over this two-year period.

The next section of the paper reviews some of the literature in this field and discusses the practitioners' methodology for evaluation of default probabilities. Section 3 presents our empirical methodology and findings, and Section 4 concludes and presents an outline of further research.

2 Review of the literature

2.1 Valuation of credit default swaps

Although credit default swaps (CDS) gained widespread attention in the recent financial crisis, they have been important financial instruments for some time, becoming "the most widely used credit derivative product..." as of 2003.[2] They serve to transfer the credit risk related to a *reference entity*, such as a sovereign borrower, from one party to another, providing the owner of the debt obligation with insurance against loss of principal value following a defined *credit event* such as bankruptcy or failure to pay scheduled coupons. In consideration, the insurer (or protection seller) receives a stream of premium payments from the debt owner, calculated from the principal of the debt instrument and an agreed *default swap spread*.

Like most derivative instruments, the CDS has zero value at its initiation, as the expected present value of premium payments is set equal to the expected payout via the market's determination of the default swap spread. However, like any derivative instrument with option-like characteristics, the mark-to-market value of the CDS during its lifetime may vary widely from zero, depending on the market's revised perception of the underlying borrower's riskiness. During the life of the CDS, the mark-to-market (MTM) value is the difference between the current market value of the remaining term's protection and the expected present value of premium payments. Both elements of MTM must take into account the probability of a credit event. In a liquid market, MTM can be expressed as the difference between the current spread (s_t) and the original spread (S_0) times a factor

[2] O'Kane and Turnbull, p. 1.

known as *Risky PV01*,[3] the expected present value of 1 bp paid as premium until default or maturity. The riskiness arises because premium payments cease if there is a credit event.

Computation of the Risky PV01 must take into account the riskiness of each premium payment by calculating the (arbitrage-free) probability of the reference entity surviving to each premium payment date. This is performed with a duration model that transforms market default swap spreads into survival probabilities by making assumptions about the underlying hazard rates. As with any duration model, for a given perceived level of risk, the probability of default is monotonically increasing with tenor. Using a model calibrated to default swap spreads, market participants compute a *cumulative probability of default*, or *cumpdf*. In this context, $(1 - cumpdf) = probability of survival$ measures the probability that no credit event will occur before maturity of the CDS. In our modeling exercise, we make use of *cumpdf* measures as a market-based evaluation of the perceived riskiness of the underlying instrument. These measures differ by sovereign borrowers, CDS tenor, and over time, providing a rich source of variation across these three dimensions of our daily data.

2.2 Models of sovereign credit risk

While the empirical models in this paper make use of the Risky PV01 and *cumpdf* measures computed from the Lehman Brothers methodology, a substantial literature has arisen describing analytical models of CDS spread pricing developed from first principles. In this section, we discuss such an approach, and contrast it with several researchers' recent efforts.

The latest contributions to the literature in this area have been focused on *reduced form models for credit risk*. These reduced form models assume that default is an unpredictable event, which can take place at any time.

In the spirit of Ang and Longstaff (2011) and Duffie and Singleton (2003), we assume that there are two sources of stochastic shocks:

- *idiosyncratic shocks* with intensity ξ: this is a sovereign specific shock that can potentially cause the default of an individual country;
- *systemic shocks* with intensity λ (with exposure γ): this is a shock that can potentially affect all the countries in the Eurozone and can explain spillover effects from one country to the other.

[3] ibid., p. 3

We assume that the interest rate and the intensities of the two types of shocks are stochastic processes characterized as follows:

$$dr = a_r (b_r - r) \cdot dt + \sigma_r \cdot \sqrt{r} \cdot dw_r \tag{1}$$

$$d\lambda = a (b - \lambda) \cdot dt + \sigma \cdot \sqrt{\lambda} \cdot dw_\lambda \tag{2}$$

$$d\xi = a_\xi (b_\xi - \xi) \cdot dt + \sigma_\xi \cdot \sqrt{\xi} \cdot dw_\xi \tag{3}$$

A CDS is an insurance (or protection) against the event of default. Consider a credit default swap written on a bond. The main assumption one needs to make is related to the portion of the underlying bond that would be recovered in case of default.

Assuming that the bond face value is 1 and the bond recovery value in case of default is Γ, [4] then the protection seller pays $(1 - \Gamma)$ at time T_k if the bond defaults during $[T_{k-1}, T_k]$. The protection buyer pays the protection fee $\overline{s} \cdot (T_k - T_{k-1})$ at time T_k in case of survival until T_k. If default takes place before T_k, then the protection buyer pays no fee at T_k or later. In this setting, the present value of the CDS to the protection buyer at time t is

$$(1 - \Gamma) \cdot \sum_{k=1}^{j} D(r, t, T_k) \cdot (P(\lambda, \xi, t, T_{k-1}) - P(\lambda, \xi, t, T_k)) - \overline{s} \cdot \sum_{k=1}^{j} V(r, \lambda, \xi, t, T_k) \cdot (T_k - T_{k-1}) \tag{4}$$

where $D(r, t, T_j)$ is the value at time t of a default-free discount bond that will pay 1 at maturity T_j. $P(\lambda, \xi, t, T_j)$ is the probability of survival over the period $[t, T_j]$ in the risk-neutral world, also termed the risk-neutral survival probability. Thus $P(\lambda, \xi, t, T_j)$ is not the real-world probability of survival, which is typically greater than $P(\lambda, \xi, t, T_j)$. Notice that we can infer $P(\lambda, \xi, t, T_j)$ once we know the bond prices $V(r, \lambda, \xi, t, T_j)$ and $D(r, t, T_j)$.

Since at initiation the CDS is worthless to either party, it follows that $\overline{s}$ is set in such a way that

$$\overline{s}_j = (1 - \Gamma) \cdot \frac{\sum_{k=1}^{j} D(r, t, T_k) \cdot (P(\lambda, \xi, t, T_{k-1}) - P(\lambda, \xi, t, T_k))}{\sum_{k=1}^{j} V(r, \lambda, \xi, t, T_k) \cdot (T_k - T_{k-1})} \tag{5}$$

[4] We will elaborate on Γ in the following two subsections.

The value at time t of a defaultable discount bond that *promises* to pay 1 at maturity T_j is

$$V(t, T_j) = E_t^* \left[e^{-\int_t^{T_j} r_\tau d\tau} \cdot 1_{x>T_j} \right] \tag{6}$$

$$1_{x>T_j} = \begin{cases} 1 \text{ if default time } x \text{ is } x > T_j, \\ 0 \text{ if default time } x \text{ is } x \leq T_j. \end{cases}$$

Here we assume that the defaultable discount bond becomes worthless in case of default. If the default time x and the default-free short interest rate r_τ are independent random variables, we can write

$$\begin{aligned} V(t, T_j) &= E_t^* \left[e^{-\int_t^{T_j} r_\tau d\tau} \cdot 1_{x>T_j} \right] \\ &= E_t^* \left[e^{-\int_t^{T_j} r_\tau d\tau} \right] \cdot E_t^* \left[1_{x>T_j} \right] \\ &= D(r, t, T_j) \cdot P(\lambda, \xi, t, T_j) \end{aligned} \tag{7}$$

Denote with $P(\lambda, \xi, t, T_j)$ the probability, calculated at the present time t, that the debtor does not default in the time interval $[t, T_j]$, with $T_j > t$, given that the debtor has not yet defaulted at time t. Thus $P(\lambda, \xi, t, T_j)$ is the probability of survival up to time T_j. We assume that, given that the debtor has not yet defaulted at T_j, the probability of default in the infinitesimally short time interval $[T_j, T_j + dt]$ is

$$(\gamma\lambda + \xi) \cdot dt$$

Notice that the event of default in any infinitesimal interval dt is independent of the event of survival/default in any other time interval. Thus, the probability of default in the time interval $[T_j, T_j + dt]$ computed at time t and given that the debtor has not yet defaulted at t, is

$$P(\lambda, \xi, t, T_j) \cdot (\gamma\lambda + \xi) \cdot dt$$

which is the probability of surviving during $[t, T_j]$ times the probability of default during $[T_j, T_j + dt]$.

It can be shown (see Appendix C) that:

$$P(\lambda, \xi, t, T_{j-1}) - P(\lambda, \xi, t, T_j) = P(\lambda, \xi, t, T_{j-1})(\gamma\lambda + \xi)\, dt$$

$$P(\lambda, \xi, t, T_{j-1})(\gamma\lambda + \xi) = -\frac{\partial P(\lambda, \xi, t, T_{j-1})}{\partial T_{j-1}}$$

When both λ and ξ are stochastic processes

$$P(\lambda, \xi, t, T_{j-1}) = e^{-\int_t^{T_{j-1}} (\gamma\lambda_s + \xi_s) ds}.$$

We will develop the CDS pricing model making two possible assumptions about the value of the bond immediately after default. Two of the main recovery assumptions we will be dealing with in our model are

- recovery of market value (RMV).
- recovery of face value (RF);

The two assumptions are described in detail in the subsections below.

2.2.1 Recovery of market value (RMV)

As RMV simplifies the mathematics considerably, it is probably the more tractable recovery assumption although it is generally less realistic than the RF assumption. RMV says that

$$V(\lambda, \xi, r, t, T_j) = (1 - L) \cdot V(\lambda, \xi, r, t, T_{j-1})$$
$$0 \leq L \leq 1$$

where:

- $V(\lambda, \xi, r, t, T_{j-1})$ is the market value of the defaultable bond just before default
- L is the loss parameter; thus the loss is a fraction of $V(\lambda, \xi, r, t, T_{j-1})$;
- j is the default time and $j - 1$ is time just before default.

Thus RMV says that the bond recovery value is a fraction $(1 - L)$ of the bond market value just before default $V(\lambda, \xi, r, t, T_{j-1})$. When $V(\lambda, \xi, r, t, T_{j-1})$ significantly differs from the bond face value, the RMV assumption becomes less precise, particularly for fixed coupon bonds of long maturity, but less so for floating rate instruments.

Under the RMV assumption a defaultable coupon bond can still be viewed and valued as a portfolio of defaultable zero coupon bonds. The RMV assumption is made also in the

valuation of "vulnerable" over-the-counter (OTC) derivatives, such as swaps and options subject to default risk.

At time t the present value of the CDS to the protection buyer is

$$\sum_{k=1}^{j}\left[1-(1-L)\cdot V\left(\lambda,\xi,r,t,T_{k-1}\right)\right]D\left(r,t,T_k\right)\cdot\left(P\left(\lambda,\xi,t,T_{k-1}\right)-P\left(\lambda,\xi,t,T_k\right)\right)$$
$$-\,\overline{s}\cdot\sum_{k=1}^{j}V\left(r,\lambda,\xi,t,T_k\right)\cdot\left(T_k-T_{k-1}\right).$$

Since at initiation the CDS is worthless to either party, it follows that $\overline{s}$ is set in such a way that

$$\overline{s}_j=\frac{\sum_{k=1}^{j}\left[1-(1-L)\cdot V\left(\lambda,\xi,r,t,T_{k-1}\right)\right]D\left(r,t,T_k\right)\cdot\left(P\left(\lambda,\xi,t,T_{k-1}\right)-P\left(\lambda,\xi,t,T_k\right)\right)}{\sum_{k=1}^{j}V\left(r,\lambda,\xi,t,T_k\right)\cdot\left(T_k-T_{k-1}\right)}.$$

where $\left(P\left(\lambda,\xi,t,T_{k-1}\right)-P\left(\lambda,\xi,t,T_k\right)\right)$ is the probability of default in the time interval $[T_{k-1},T_k]$. Notice that in this case, we are making the assumption that the default event can only happen once a year and we haven't made any specific assumption about the frequency of the payment of the premia on the CDS.

One of the models that we plan to estimate follows the steps of Duffie and Singleton (1999), Pan and Singleton (2008), Duffie (2011) and Longstaff et al. (2011). This model makes the Recovery Market Value (RMV) assumption: at default time j the bond value drops to $(1-L)\,V\left(\lambda,\xi,r,t,T_{j-1}\right)$. The model by Duffie and Singleton is very tractable and popular. The main difference in our setting is that, unlike in Duffie and Singleton, there are two sources of stochastic shocks[5]

Define

$$\psi=\sqrt{a^2+2\sigma^2}$$

$$A\left(t,T_j,a,b,\sigma\right)=\ln\left[\left(\frac{2\psi e^{(\psi+a)\left(T_j-t\right)/2}}{(\psi+a)\left(e^{\psi\left(T_j-t\right)}-1\right)+2\psi}\right)^{\frac{2ab}{\sigma^2}}\right].$$

$$B\left(t,T_j,a,b,\sigma\right)=\frac{2\left(e^{\psi\left(T_j-t\right)}-1\right)}{(\psi+a)\left(e^{\psi\left(T_j-t\right)}-1\right)+2\psi}$$

[5]See Appendix A for details about the properties of this model.

$$
\begin{aligned}
D\left(r,t,T_j\right) &= E_t^*\left[e^{-\int_t^{T_j} r_\tau d\tau}\right] \\
&= e^{A(t,T_j,a_r,b_r,\sigma_r)-r\cdot B(t,T_j,a_r,b_r,\sigma_r)}
\end{aligned}
$$

$$
\begin{aligned}
P\left(\lambda,\xi,t,T_j\right) &= E_t^*\left[e^{-\int_t^{T_j}(\gamma\lambda_\tau+\xi_\tau)\cdot L\cdot d\tau}\right] \\
&= e^{A\left(t,T_j,\gamma a,L\gamma b,\sqrt{L}\gamma\sigma\right)-\lambda\cdot\gamma\cdot L\cdot B\left(t,T_j,\gamma a,L\gamma b,\sqrt{L}\gamma\sigma\right)} \\
&\cdot e^{A\left(t,T_j,a_\xi,Lb_\xi,\sqrt{L}\sigma_\xi\right)-\xi\cdot L\cdot B\left(t,T_j,a_\xi,Lb_\xi,\sqrt{L}\sigma_\xi\right)}
\end{aligned}
$$

$$
m\arg inalpdf\left(T_j\right) = P\left(\lambda,\xi,t,T_{j-1}\right) - P\left(\lambda,\xi,t,T_j\right)
$$

This is the probability of default in $[T_{j-1},T_j]$ given that no default has occurred until T_{j-1}.

While in the Lehman Brothers dataset the cumulative probability of default is defined under the assumption that the instantaneous probability of default ξ (probability that default occurs in the infinitesimally short time interval dt)is a constant, in our model cumpdf is more general because ξ is a stochastic process:

$$
\begin{aligned}
cumpdf\left(x \le T_j\right) &= 1 - P\left(\lambda,\xi,t,T_j\right) \\
&= \sum_{k=1}^{j}\left(P\left(\lambda,\xi,t,t+(k-1)\right) - P\left(\lambda,\xi,t,t+k\right)\right)
\end{aligned}
$$

where $T_j \equiv t+j$.

This is the probability that the default event x has occurred by the end of year T_j.

$$
\begin{aligned}
V\left(\lambda,\xi,r,t,T_j\right) &= E_t^*\left[e^{-\int_t^{T_j}(r_\tau+(\gamma\lambda_\tau+\xi_\tau)\cdot L)\cdot d\tau}\right] \\
&= D\left(r,t,T_j\right)\cdot P\left(\lambda,\xi,t,T_j\right) \\
&= E_t^*\left[e^{-\int_t^{T_j} r_\tau\cdot d\tau}\right]\cdot E_t^*\left[e^{-\int_t^{T_j}(\gamma\lambda_\tau+\xi_\tau)\cdot L\cdot d\tau}\right]
\end{aligned}
$$

$$
\begin{aligned}
V(\lambda,\xi,r,t,T_j) &= E_t^*\left[e^{-\int_t^{T_j} r_\tau \cdot d\tau}\right] \cdot E_t^*\left[e^{-\int_t^{T_j}(\gamma\lambda_\tau+\xi_\tau)\cdot L\cdot d\tau}\right] \\
&= e^{A(t,T_j,a_r,b_r,\sigma_r)-r\cdot B(t,T_j,a_r,b_r,\sigma_r)} \\
&\cdot e^{A\left(t,T_j,\gamma a,L\gamma b,\sqrt{L}\gamma\sigma\right)-\lambda\cdot\gamma\cdot L\cdot B\left(t,T_j,\gamma a,L\gamma b,\sqrt{L}\gamma\sigma\right)} \\
&\cdot e^{A\left(t,T_j,a_\xi,Lb_\xi,\sqrt{L}\sigma_\xi\right)-\xi\cdot L\cdot B\left(t,T_j,a_\xi,Lb_\xi,\sqrt{L}\sigma_\xi\right)}
\end{aligned}
$$

Single issuer case The CDS spread for the single issuer case is given by $\overline{s}_{j,t}$ which is the model value of the CDS price whose quotation is taken on day t, $t = 1, 2, ...M$ and whose tenor is j (time to maturity $j = T_j - t$ where T_j are the maturity dates), $j = 1, 2, ...J$.

The CDS give the holder protection against default of the underlying asset.

- Without loss of accuracy, we assume that the credit event (default) can occur daily.
- Consider: $T_j \equiv t + j$
- The protection seller pays $(1-(1-L)V(\lambda,\xi,r,t,T_{j-1}))$ if default happens between T_{j-1} and T_j
- We assume that the CDS pays premia continuously (daily).

Single Issuer case

The CDS spread for the single issuer case is given by:

$$
\overline{s}_{j,t} = \frac{\sum_{k=1}^{j}[1-(1-L)V(\lambda,\xi,r,t,t+(k-1))]D(r,t,t+k)\cdot(P(\lambda,\xi,t,t+(k-1))-P(\lambda,\xi,t,t+k))}{\sum_{k=1}^{j}V(\lambda,\xi,r,t,t+k)}
$$

$$
\min_{\mathbf{a},\mathbf{b},\sigma,\mathbf{a}_r,b_r\sigma_r,\mathbf{a}_\xi,b_\xi,\sigma_\xi,\gamma} \sum_{j=1}^{J}\sum_{t=1}^{M}\left[s_{j,t}-\overline{s}_{i,t}\left(\mathbf{a},\mathbf{b},\sigma,\mathbf{a}_r,b_r,\sigma_r,\mathbf{a}_\xi,b_\xi,\sigma_\xi,\gamma\right)\right]^2
$$

Multi-issuer case

In the case where we have $i = 1, 2, ...N$ possible issuers, the CDS spread becomes:

$$
\overline{s}_{i,j,t} = \frac{\sum_{k=1}^{j}[1-(1-L_i)V_i(\lambda,\xi,r,t,t+(k-1))]D(r,t,t+k)\cdot(P_i(\lambda,\xi,t,t+(k-1))-P_i(\lambda,\xi,t,t+k))}{\sum_{k=1}^{j}V_i(\lambda,\xi,r,t,t+k)}
$$

and the minimization problem would become:

$$\min_{\mathbf{a},\mathbf{b},\sigma,\mathbf{a}_r,b_r\sigma_r,[\mathbf{a}_\xi],[b_\xi],[\sigma_\xi],\gamma} \sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{t=1}^{M} \left[s_{i,j,t} - \overline{s}_{i,i,t}\left(\mathbf{a},\mathbf{b},\sigma,\mathbf{a}_r,b_r,\sigma_r,[\mathbf{a}_\xi],[b_\xi],[\sigma_\xi],\gamma\right)\right]^2$$

where

$$[\widehat{\mathbf{a}}_\xi] = \begin{bmatrix} \widehat{\mathbf{a}}_{\xi_1} \\ \widehat{\mathbf{a}}_{\xi_2} \\ . \\ . \\ \\ \\ \widehat{\mathbf{a}}_{\xi_{10}} \end{bmatrix}$$

$$\left[\widehat{\mathbf{b}}_\xi\right] = \begin{bmatrix} \widehat{\mathbf{b}}_{\xi_1} \\ \widehat{\mathbf{b}}_{\xi_2} \\ . \\ . \\ \\ \\ \widehat{\mathbf{b}}_{\xi_N} \end{bmatrix}$$

$$[\widehat{\sigma}_\xi] = \begin{bmatrix} \widehat{\sigma}_{\xi_1} \\ \widehat{\sigma}_{\xi_2} \\ . \\ . \\ \\ \\ \widehat{\sigma}_{\xi_N} \end{bmatrix}$$

2.2.2 Recovery of face value (RF)

RF is normally the most realistic assumption. Assuming that the bond face value is 1, RF says that the bond recovery value at the default time x is

$$V(r,\lambda,\xi,t,x) = R$$
$$0 \leq R \leq 1$$

i.e. the recovery value R is a fraction of the bond face value, which is equal to 1. RF is realistic but complicates a bit the mathematics of the valuation models.

At time t the present value of the CDS to the protection buyer is

$$(1-R)\cdot\sum_{k=1}^{j} D\left(r,t,T_k\right)\cdot\left(P\left(\lambda,\xi,t,T_{k-1}\right)-P\left(\lambda,\xi,t,T_k\right)\right)-\overline{s}\cdot\sum_{k=1}^{j} V\left(r,\lambda,\xi,t,T_k\right)\cdot\left(T_k-T_{k-1}\right).$$

Since at initiation the CDS is worthless to either party, it follows that $\overline{s}$ is set in such a way that

$$\overline{s}_j=(1-R)\cdot\frac{\sum_{k=1}^{j} D\left(r,t,T_k\right)\cdot\left(P\left(\lambda,\xi,t,T_{k-1}\right)-P\left(\lambda,\xi,t,T_k\right)\right)}{\sum_{k=1}^{j} V\left(r,\lambda,\xi,t,T_k\right)\cdot\left(T_k-T_{k-1}\right)}.$$

where $\left(P\left(\lambda,\xi,t,T_{k-1}\right)-P\left(\lambda,\xi,t,T_k\right)\right)$ is the probability of default in the time interval $[t_k,t_{k+1}]$.

$$B\left(t,T_j,a,b,\sigma\right)=\frac{2\left(e^{\psi(T_j-t)}-1\right)}{(\psi+a)\left(e^{\psi(T_j-t)}-1\right)+2\psi}$$

$$\psi=\sqrt{a^2+2\sigma^2}$$

$$A\left(t,T_j,a,b,\sigma\right)=\ln\left(\frac{2\psi e^{(\psi+a)(T_j-t)/2}}{(\psi+a)\left(e^{\psi(T_j-t)}-1\right)+2\psi}\right)^{\frac{2ab}{\sigma^2}}$$

$$\begin{aligned}V\left(\lambda,\xi,r,t,T_j\right)&=D\left(r,t,T_j\right)P\left(\lambda,\xi,t,T_j\right)\\&=E_t^*\left[e^{-\int_t^{T_j} r_\tau\cdot d\tau}\right]\cdot E_t^*\left[e^{-\int_t^{T_j}\gamma\lambda_\tau\cdot d\tau}\right]E_t^*\left[e^{-\int_t^{T_j}\xi_\tau\cdot d\tau}\right]\\&=e^{A(t,T_j,a_r,b_r,\sigma_r)-r\cdot B(t,T_j,a_r,b_r,\sigma_r)}\cdot e^{A(t,T_j,a,b,\sigma)-\gamma\lambda\cdot B(t,T_j,a,b,\sigma)}\\&\cdot e^{A\left(t,T_j,a_\xi,b_\xi,\sigma_\xi\right)-\xi\cdot B\left(t,T_j,a_\xi,b_\xi,\sigma_\xi\right)}\end{aligned}$$

It can be shown (see Appendix D for the details) that:

$$
\begin{aligned}
\left(P\left(\lambda,\xi,t,T_{j-1}\right)-P\left(\lambda,\xi,t,T_{j}\right)\right) &= -\frac{\partial P\left(\lambda,\xi,t,T_{j-1}\right)}{\partial T_{j-1}} \\
&= \left(-\frac{\partial A\left(t,T_{j-1},a,b,\sigma\right)}{\partial T_{j-1}}+\gamma\lambda\frac{\partial B\left(t,T_{j-1},a,b,\sigma\right)}{\partial T_{j-1}}\right)P\left(\lambda,\xi,t,T_{j-1}\right) \\
&+\left(-\frac{\partial A\left(t,T_{j-1},a_{\xi},b_{\xi},\sigma_{\xi}\right)}{\partial T_{j-1}}+\xi\frac{\partial B\left(t,T_{j-1},a_{\xi},b_{\xi},\sigma_{\xi}\right)}{\partial T_{j-1}}\right)P\left(\lambda,\xi,t,T_{j-1}\right)
\end{aligned}
$$

where

$$
P\left(\lambda,\xi,t,T_{j}\right)=e^{A(t,T_j,a,b,\sigma)-\gamma\lambda\cdot B(t,T_j,a,b,\sigma)}\cdot e^{A\left(t,T_j,a_{\xi},b_{\xi},\sigma_{\xi}\right)-\xi\cdot B\left(t,T_j,a_{\xi},b_{\xi},\sigma_{\xi}\right)}
$$

$$
\begin{aligned}
\frac{\partial B\left(t,T_{j-1},a,b,\sigma\right)}{\partial T_{j-1}} &= -\frac{1}{2}\sigma^2B^2\left(t,T_{j-1},a,b,\sigma\right)-aB\left(t,T_{j-1},a,b,\sigma\right)+1 \\
\frac{\partial A\left(t,T_{j-1},a,b,\sigma\right)}{\partial T_{j-1}} &= -abB\left(t,T_{j-1},a,b,\sigma\right)
\end{aligned}
$$

$$
\begin{aligned}
m\arg inalpdf\left(T_j\right) &= P\left(\lambda,\xi,t,T_{j-1}\right)-P\left(\lambda,\xi,t,T_{j}\right) \\
&= -\frac{\partial P\left(\lambda,\xi,t,T_{j-1}\right)}{\partial T_{j-1}}
\end{aligned}
$$

This is the probability of default in $[T_{j-1},T_j]$ given that no default has occurred until T_{j-1}.

While in the Lehman Brothers dataset the cumulative probability of default is defined under the assumption that the instantaneous probability of default ξ (probability that default occurs in the infinitesimally short time interval dt), in our model cumpdf is a more general variable defined in the following way:

$$
\begin{aligned}
cumpdf\left(x\leq T_j\right) &= 1-P\left(\lambda,\xi,t,T_j\right) \\
&= \sum_{k=1}^{j}\left(P\left(\lambda,\xi,t,t+(k-1)\right)-P\left(\lambda,\xi,t,t+k\right)\right) \\
&= -\sum_{k=1}^{j}\frac{\partial P\left(\lambda,\xi,t,T_{k-1}\right)}{\partial T_{k-1}}
\end{aligned}
$$

where $T_j\equiv t+j$.

This is the probability that the default event x has occurred by the end of year T_j.

Considering annual quotations for bonds with maturity j :

$$\overline{s}_{j,t} = (1-R) \cdot \frac{\sum_{k=1}^{j} D(r,t,t+k) \cdot (P(\lambda,\xi,t,t+(k-1)) - P(\lambda,\xi,t,t+k))}{\sum_{k=1}^{j} V(\lambda,\xi,r,t,t+k)}$$

where $(P(\lambda,\xi,t,T_{k-1}) - P(\lambda,\xi,t,T_k))$ is the risk-neutral probability of default for infinitesimal period $[T_{k-1},T_k]$ given that no default has occurred at T_{k-1} where $T_k \equiv t+k$.

$$\overline{s}_{j,t} = (1-R) \cdot \frac{\sum_{k=1}^{j} D(r,t,t+k) \cdot \left(-\frac{\partial P(\lambda,\xi,t,t+(k-1))}{\partial(t+(k-1))}\right)}{\sum_{k=1}^{j} V(\lambda,\xi,r,t,t+k)}$$

Single issuer case

The CDS spread for the single issuer case is given by $\overline{s}_{j,t}$ which is the model value of the CDS price whose quotation is taken on day t, $t = 1,2,...M$ and whose tenor is j (time to maturity $j = T_j - t$ where T_j are the maturity dates), $j = 1,2,....J$.

The CDS give the holder protection against default of the underlying asset.

- Without loss of accuracy, we assume that the credit event (default) can occur daily.
- Consider: $T_j \equiv t+j$
- The protection seller pays $(1-R)$ if default happens between T_{j-1} and T_j where the probability of default in $[T_{j-1},T_j]$ is: $[(P(\lambda,\xi,t,T_{j-1}) - P(\lambda,\xi,t,T_j))]$.
- We assume that the CDS pays daily premia.

The CDS spread for the single issuer case is given by:

$$\min_{\mathbf{a},\mathbf{b},\sigma,\mathbf{a}_r,b_r\sigma_r,\mathbf{a}_\xi,b_\xi,\sigma_\xi,\gamma} \sum_{j=1}^{J}\sum_{t=1}^{M} \left[s_{j,t} - \overline{s}_{i,t}(\mathbf{a},\mathbf{b},\sigma,\mathbf{a}_r,b_r,\sigma_r,\mathbf{a}_\xi,b_\xi,\sigma_\xi,\gamma)\right]^2 \tag{8}$$

- where for a fixed issuer, $\overline{s}_{j,t}$ is the model value of the CDS price whose quotation is taken on day k for $k = 1,2,...M$ and whose tenor is $j = 1,2,....J$. The model price depends on $\mathbf{a},\mathbf{b},\sigma,\mathbf{a}_r,b_r\sigma_r,\mathbf{a}_\xi,b_\xi,\sigma_\xi,\gamma$;

Multi-issuer case

In this case, the CDS spread is

$$\overline{s}_{i,j,t} = (1 - R_i) \cdot \frac{\sum_{k=1}^{j} D\left(r, t, t+k\right) \cdot \left(-\frac{\partial P_i(\lambda,\xi,t,t+(k-1))}{\partial(t+(k-1))}\right)}{\sum_{k=1}^{j} V_i\left(\lambda, \xi, r, t, t+k\right)}$$

and the minimization becomes

$$\min_{\mathbf{a},\mathbf{b},\sigma,\mathbf{a}_r,b_r\sigma_r,[\mathbf{a}_\xi],[b_\xi],[\sigma_\xi],\gamma} \sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{t=1}^{M} \left[s_{i,j,t} - \overline{s}_{i,i,t}\left(\mathbf{a}, \mathbf{b}, \sigma, \mathbf{a}_r, b_r, \sigma_r, [\mathbf{a}_\xi], [b_\xi], [\sigma_\xi], \gamma\right)\right]^2$$

3 Empirical methodology and findings

3.1 Descriptive measures

Our dataset includes daily quotes for one- through ten-year CDS spreads of sovereign borrowers over all trading days of 2009–2010. We focus on eleven countries in the Eurozone: Austria, Belgium, Germany, Spain, Finland, France, Greece, Ireland, Italy, Netherlands and Portugal. Our descriptive statistics focus on the one-year and five-year spreads due to their liquidity, although we make use of all available tenors in the modeling exercise. Table 1 presents summary statistics for the one-year spreads, while Table 2 presents statistics for the five-year spreads. As is evident in both tables, five countries (Spain, Greece, Ireland, Italy and Portugal) have markedly higher spreads over this turbulent period, and all but Italy also exhibit much greater volatility in the levels of spreads, measured by their standard deviation (sd).

As Germany has been considered the anchor for Eurozone sovereign debt, it is illuminating to evaluate CDS spreads relative to the contemporaneous German spread. In Tables 3 and 4, we provide the descriptive statistics for one- and five-year relative spreads, respectively, for the other ten sovereign borrowers. During this period, Finland, France and the Netherlands exhibit relatively small spreads relative to Germany, with Austria and Belgium considerably higher. Those three countries also have much lower volatility of their relative spreads. During this period, market perceptions of Italian debt are converging on that of the PIGS (Portugal, Ireland, Greece and Spain), with the median Italian relative spread higher than that of Spain.

In evaluating the spillover effects across borrowers, it is instructive to consider the correlations of changes in their spreads. Tables 5 and 6 present these correlations for the full sample period. As is evident, there are sizable positive correlations for both tenors, indicating that even the most creditworthy borrowers are likely to experience some market adjustments in their spreads when riskier borrowers' spreads increase. At opposite ends of the spectrum, changes in Greek (GRC) and German (DEU) spreads are correlated +0.27 at the one-year tenor and +0.46 at the five-year tenor. An evaluation of the principal components of these eleven borrowers' spread changes shows that the first principal component explains 50 per cent of the variation in the entire set (and the first two, 65 per cent) for the one-year tenor, and 66 per cent (79 per cent) for the five-year tenor.

These correlations, computed for the full sample period, may not tell the whole story. The linkages between sovereign borrowers' perceived risk may vary considerably over time as political and economic circumstances change. Thus, we have computed moving-window correlations, using a window of 21 trading days (roughly a month) for the five riskiest borrowers' CDS spread changes at the five-year tenor. Figures 1 and 2 illustrate some of these correlations for Ireland and Greece, respectively.

Changes in the Irish spreads are highly correlated (+0.8 or more) with changes in the other four risky borrowers' spreads for the first eight months of 2009, falling to half that value in September 2009. Closer correlations return until the onset of the Greek crisis in May 2010, when changes in the Greek spread become very weakly correlated with those of other borrowers. This repeats in early fall 2010. For the latter half of 2010, correlations of Irish spreads' changes with those of the other risky borrowers are generally lower than at the beginning of the sample.

From the perspective of the Greek spreads (Figure 2), correlations with other borrowers' spread changes persist though mid-2010, with the exception of the Irish episode in late 2009. These correlation patterns then deteriorate considerably in June 2010, and become much more variable in the latter half of that year.

3.1.1 Moving-window volatility estimates

Another focus of interest might be the volatility exhibited by these spreads, as for a given borrower and tenor, that reflects market participants' uncertainty about the riskiness of the underlying sovereign debt. Accordingly, we have computed moving-window standard deviations of the CDS quote series for each borrower and tenor. In Figure 3, we illustrate

the moving-window volatility estimates (using a 22-day window) for the one-year tenor. The upper panel shows correlated increases in the volatility of French and Italian CDS spreads during spring 2010, with an earlier peak in February. That peak is echoed by Greek spreads, which reach a much higher level of volatility in the spring of 2010.

Figure 4 compares the same borrowers' spreads for the five-year tenor. At the longer tenor, higher volatility of French and Italian CDS spreads persists through the latter half of 2010. In both panels, the marked increase in uncertainty in early 2010 relative to 2009 is evident.

3.1.2 Cross-sectional volatility estimates

While the moving-window estimates reflect the market's uncertainty about a specific borrower and tenor, we might also consider the cross-sectional volatility present at each point in time across borrowers. This abstracts from the average level of CDS spreads for each trading day, and considers the degree to which troubled borrowers' spreads are diverging or converging with those of less risky borrowers. These cross-sectional volatility measures are graphed in Figure 5 for one-year and five-year tenors.

The patterns of the moving-window volatility estimates are to some degree reflected in the cross-sectional volatility measures. For most of 2009, volatility is low at both the one-year and five-year tenors, increasing in February 2010 and then increasing sharply through the rest of the year.

Although these descriptive measures are illuminating, they only provide evidence of comovements, representing spillover effects across sovereign borrowers. We turn now to an econometric modeling strategy, using the market-based measure of the cumulative probability of default, which formalizes these interlinkages and provides the basis for a short-term forecasting exercise.

3.2 Modeling the evolution of sovereign CDS spreads

For each of the 11 sovereign borrowers in our sample, we have daily quotations on their CDS spread for tenors of 1, 2, ... 10 years. We model the vector of quotes for a given issuer and quote date, using a seemingly unrelated regression (SUR) approach where each issuer has its own equation. This permits heterogeneity across issuers to be reflected in not only the intercept of that equation (e.g., via fixed effects) but allows for separate

vectors of coefficients on the explanatory variables, and differing error variances, across issuers. The rationale for the SUR technique is the potential contemporaneous correlation of errors across equations (issuers), which is highly relevant in this modeling context in both economic and econometric terms.

The explanatory variables used in each equation include the previous trading day's *cumpdf* value for the country itself (the own effect) and a set of lagged *cumpdf* values for each of the other countries. As the data for each equation include tenors of one to ten years, separate intercepts are introduced by tenor, and the tenor indicators are interacted with the own-country lagged *cumpdf*, allowing the effect of changes in the expected probability of default to have differing effects on the various tenors. The estimated equation for each country included in the SUR is thus:

$$Q_{i,\tau,t} = \beta_0 + \beta_1 cumpdf_{i,t-1} + \sum_{q=2}^{10} \gamma_q \tau_q + \sum_{j=2}^{10} \beta_j \tau_j cumpdf_{i,t-1} + \sum_{k \in K^*} \delta_k cumpdf_{k,t-1} + \epsilon_{i,\tau,t} \quad (9)$$

where $Q_{i,\tau,t}$ is the CDS spread for borrower i, tenor τ, on day t; $cumpdf_{i,t-1}$ is the previous trading day's expected probability of default for borrower i; and $cumpdf_{k,t-1}$ is the corresponding measure for borrower $k \neq i$, such that the set K^* includes all borrowers except country i. Thus, the coefficient β_0 is the intercept for the one-year CDS spread, while $(\beta_0 + \gamma_q)$ is the intercept for the q-year spread. The coefficient β_1 expresses the sensitivity of the current one-year CDS spread to the most recently computed probability of that borrower's default, while $\beta_2 \ldots \beta_{10}$ express the differentials in that sensitivity for CDS spreads for 2, ..., 10-year tenors. The coefficients δ_k measure the spillover effects between other sovereign borrowers' expected probability of default and borrower i's spread.

3.3 Estimation for the full sample period

We first consider estimating the SUR for the eleven sovereign borrowers over the full sample of all trading days in 2009–2010. Summary statistics for the full-sample estimation are given in Table 7. Although all equations fit quite well, the RMSE (root mean square error) values, expressed in basis points, vary widely over borrowers from 1.7–2.8 bp for the most creditworthy borrowers to 14–15 bp for Ireland and Portugal and 38 bp for Greece. The Breusch–Pagan test for a diagonal residual covariance matrix overwhelmingly rejects

that null hypothesis, suggesting that there are strong contemporaneous correlations across borrowers' disturbance terms, as we would expect from theory.

Although it would hardly be useful to review all 319 estimated coefficients from this model, several sets in particular are of special interest. First, we consider the effect of changes in the expected probability of default over CDS spread tenors. These can be computed for each borrower by summing β_1 and the $\beta_2 \ldots \beta_{10}$ coefficients. The patterns of these effects are quite similar over borrowers, so we present only the German (DEU) coefficients as a representative sample in Figure 6. [6] It is evident that the impact of changes in the expected probability of default are greatest (about 52 bp) on the one-year CDS spread, with the two-year tenor increasing by about half as much (about 27 bp), and longer tenors exhibiting even lower sensitivities.

Second, let us consider the spillover effects from changes in the German (DEU) expected probability of default on those of the most troubled borrowers: the PIIGS (Portugal, Ireland, Italy, Greece, Spain). These effects, expressed by the *cumpdfDEU* coefficient in the respective borrowers' equations, are displayed in Table 8. These effects are significant for three of the five borrowers, positive for Ireland and Italy but negative and significant for Greece. Thus, there is evidence for spillovers between the expected probability of default of the most creditworthy borrower, Germany, and the CDS spreads of the most troubled borrowers.

Third, we consider the opposite effects: those gauging the impact of changes in the expected probability of default among the PIIGS on German CDS spreads. We do this for both the five PIIGS and the three least creditworthy at the time: Greece, Ireland and Portugal. We compute the linear combination of the coefficients of these borrowers' $cumpdf_{t-1}$ on German CDS spreads, in point and interval form. For the full set of five PIIGS, that summary coefficient is -0.0617 with a standard error of 0.0313 and p-val of 0.049, suggesting that a parallel increase in all PIIGS' expected probability of default would be expected to lower German spreads. However, when we consider only Greece, Ireland, and Portugal, the summary coefficient is 0.0363 with a standard error of 0.0143 and p-val of 0.011, suggesting that increases in the expected probability of default of those borrowers would be associated with higher German CDS spreads.

[6] These effects are estimated so precisely that the confidence interval bands are not visible in the figure.

3.4 Estimation for subperiods with *ex ante* forecasts

In this section, we report the results for three forecasting exercises. In the first exercise we reestimate the model over the first eight months (January–August 2009) and compute static forecasts through the following month (trading days 1 September–2 October 2009). In the second, we reestimate the model over the first 16 months, with the sample ending on 30 April 2010, with forecasts performed for 3 May–2 June 2010. Finally, we reestimate the model for the first 23 months, ending on 26 November 2010 and forecasting for 29 November–29 December 2010.

For each of these three estimation periods, we compute four forecast statistics for each tenor and borrower: RMSE, the root mean square error of forecast; MAE, the mean absolute error of forecast; MAPE, the mean absolute percentage error of forecast; and Theil's U statistic. These statistics are computed for every tenor and borrower. We focus on the five PIIGS borrowers (Portugal, Ireland, Italy, Greece and Spain) and Germany, and present results for 1-, 3-, and 5-year tenors.

For the first estimation period, using only eight months of data, we present the forecast error statistics in Table 9. In that period, covering forecasts for September 2009, it is evident that forecasts for Greece and Ireland are much less accurate, on the basis of RMSE or MAE, than those of the other PIIGS and Germany. This ranking does not hold for the MAPE nor Theil's U measures. The German RMSE is roughly constant over tenors, while forecasts for Greece and Ireland at short tenors are much less accurate than those at longer tenors.

For the second estimation period, in which 16 months of data are used, we present the forecast error statistics in Table 10 relating to forecasts of May 2010. Focusing on RMSE and MAE, the forecast errors are an order of magnitude larger than in the first period, while scaling by MAPE or Theil's U removes this effect. These magnitudes do not appear to be sensitive to tenors as in the first period.

For the third estimation period, in which 23 months of data are used, we present the forecast error statistics in Table 11 for forecasts of December 2010. The RMSE, MAE and MAPE figures are smaller than those in the second period for the most troubled borrowers. The forecast accuracy of German CDS spreads does not differ widely over the three estimation periods, whereas considerable deterioration is evident in the troubled borrowers' forecast statistics for the second and third periods.

4 Conclusions

This analysis of high-frequency data on the CDS spreads of sovereign borrowers illustrates the advantages of working with more detailed information from these markets. Many features of the data appear at the daily frequency that are masked, or difficult to estimate, at the lower weekly or monthly frequencies considered in most empirical research. The availability of the entire term structure of CDS quotes also allows us to consider how short-term 'insurance policies' behave relative to those written for longer tenors.

Our forecasting exercise illustrates that the volatility inherent in very risky borrowers' CDS spreads may make it very difficult to produce reasonable predictions, even for short horizons. On the other hand, forecasting the CDS spreads of more creditworthy borrowers is considerably more successful.

Appendix A

The Duffie and Singleton (1999) model in case of multiple sources of risk (see Ang and Longstaff) implies that the market value of the defaultable bond, in a no-arbitrage setting, needs to satisfy the following conditions:

$$
\begin{aligned}
0 = & \frac{\partial V}{\partial t} + \frac{\partial^2 V}{\partial \lambda \partial r} \rho_1 \sigma_r (r,t) \sigma (\lambda, t) + \frac{\partial^2 V}{\partial \xi \partial r} \rho_2 \sigma_r (r,t) \sigma_\xi (\xi, t) && (10) \\
& + \frac{\partial^2 V}{\partial \lambda^2} \frac{\sigma (\lambda, t)^2}{2} + \frac{\partial^2 V}{\partial \xi^2} \frac{\sigma_\xi (\xi, t)^2}{2} + \frac{\partial^2 V}{\partial r^2} \frac{\sigma_r (r,t)^2}{2} \\
& + \frac{\partial V}{\partial \lambda} m^* (\lambda, t) + \frac{\partial V}{\partial \xi} m^* (\xi, t) + \frac{\partial V}{\partial r} m^* (r,t) \\
& - (r + \gamma \lambda L) V + \frac{\partial V}{\partial r} m^* (r,t) - (r + \xi L) V && (11)
\end{aligned}
$$

$$
\begin{aligned}
& \lim_{\lambda \to \infty} V (\lambda, \xi, r, t, T) \to 0 \\
\lim_{\xi \to \infty} V (\lambda, \xi, r, t, T) \to 0 && (12)
\end{aligned}
$$

$$
\begin{aligned}
& \lim_{\lambda \to 0} V (\lambda, \xi, r, t, T) \to D (t, T) \\
\lim_{\xi \to 0} V (\lambda, \xi, r, t, T) \to D (t, T) && (13) \\
& \lim_{r \to \infty} V (\lambda, \xi, r, t, T) \to 0 \\
& V (\lambda, \xi, r, T, T) = 1.
\end{aligned}
$$

The lower boundary condition for r is omitted since it depends on the specific process of r. Notice that the upper boundary condition for $\lambda \to \infty$ is now different from the upper boundary condition $\lim_{\lambda \to \infty} V (\lambda, \xi, r, t) \to R$ imposed to PDE 19. Using the Feynman-Kac theorem we can deduce that PDE 10 implies

$$
V (\lambda, \xi, r, t, T) = E_t^* \left[e^{- \int_t^T (r_\tau + (\gamma \lambda_\tau + \xi_\tau) \cdot L) \cdot d\tau} \right].
$$

This equation shows that, under the RMV assumption, we just need to add $\lambda_\tau \cdot L$ to r_τ to take default risk into account. A special case is one whereby both r and λ follow Cox-Ingersoll-Ross CIR processes in the risk-neutral world, so that

$$
dr = a_r (b_r - r) \cdot dt + \sigma_r \cdot \sqrt{r} \cdot dw_r \tag{14}
$$

$$
d\lambda = a (b - \lambda) \cdot dt + \sigma \cdot \sqrt{\lambda} \cdot dw_\lambda \tag{15}
$$

$$
d\xi = a_\xi (b_\xi - \xi) \cdot dt + \sigma_\xi \cdot \sqrt{\xi} \cdot dw_\xi \tag{16}
$$

with $dw_r \cdot dw_\lambda = \rho_1 dt$ and $dw_r \cdot dw_\xi = \rho_2 dt$. The CIR assumption leads to a convenient closed form solution to equation 10 if we assume that r is not correlated with λ and ξ, i.e. if $\rho_1 = \rho_2 = 0$. Closed form solutions provide understanding, are easy to compute and make it easy to calibrate the model to market prices or to estimate and test the model through maximum likelihood, GMM or other econometric techniques. In other words closed form solutions for bond values make it easy to determine the values of the parameters $a_r, b_r, \sigma_r, a, b, \sigma, a_\xi, b_\xi, \sigma_\xi, \gamma$ such that the bond values predicted by the model are close to the bond prices in the market.

Under the CIR assumption, the RMV assumption and if $\rho_1 = \rho_2 = 0$, PDE 10 implies

$$V(\lambda, \xi, r, t, T) = E_t^* \left[e^{-\int_t^T r_\tau \cdot d\tau} \right] \cdot E_t^* \left[e^{-\int_t^T (\gamma\lambda_\tau + \xi_\tau) \cdot L \cdot d\tau} \right] \tag{17}$$

$$\begin{aligned} &= e^{A(t,T,a_r,b_r,\sigma_r) - r \cdot B(t,T,a_r,b_r,\sigma_r)} \\ &\cdot e^{A\left(t,T,\gamma a, L\gamma b, \sqrt{L}\gamma\sigma\right) - \lambda \cdot \gamma \cdot L \cdot B\left(t,T,\gamma a, L\gamma b, \sqrt{L}\gamma\sigma\right)} \\ &\cdot e^{A\left(t,T,a_\xi, Lb_\xi, \sqrt{L}\sigma_\xi\right) - \xi \cdot L \cdot B\left(t,T,a_\xi, Lb_\xi, \sqrt{L}\sigma_\xi\right)} \end{aligned} \tag{18}$$

with

$B(t, T, a, b, \sigma) = \frac{2\left(e^{\psi(T-t)}-1\right)}{(\psi+a)\left(e^{\psi(T-t)}-1\right)+2\psi}$

$\psi = \sqrt{a^2 + 2\sigma^2}$

$A(t, T, a, b, \sigma) = \ln\left(\frac{2\psi e^{(\psi+a)(T-t)/2}}{(\psi+a)\left(e^{\psi(T-t)}-1\right)+2\psi} \right)^{\frac{2ab}{\sigma^2}}$.

Notice that $B(t, T, a_r, b_r, \sigma_r)$ and $B\left(t, T, a, Lb, \sqrt{L}\sigma\right)$ here denote solutions to Riccati equations, not bond prices. The result in formula 17 is due to the fact that, if λ and ξ follow a CIR process, then also $(\gamma\lambda + \xi) L$ follows a CIR process. In fact $(\gamma\lambda + \xi) L$ (where L is constant) by Ito's lemma follows the process

$$\begin{aligned} d((\gamma\lambda + \xi) L) &= L \cdot (\gamma d\lambda + d\xi) \\ &= L \cdot \left(\gamma a (b - \lambda) \cdot dt + \gamma\sigma\sqrt{\lambda} \cdot dw_\lambda + a_\xi (b_\xi - \xi) \cdot dt + \sigma_\xi \sqrt{\xi} \cdot dw_\xi \right) \end{aligned}$$

Appendix B

<u>Euler "full truncation" discretization scheme</u>

$$
\begin{aligned}
dr &= a_r \left(b_r - r\right) \cdot dt + \sigma_r \cdot \sqrt{r} \cdot dw_r \\
r\left(t + dt\right) &= r\left(t\right) + a_r \left(b_r - r\left(t\right)^+\right) \cdot dt + \sigma_r \cdot \sqrt{r\left(t\right)^+} \cdot dw_r \\
r\left(t\right)^+ &\equiv Max\left(r\left(t\right), 0\right) \\
d\lambda &= a\left(b - \lambda\right) \cdot dt + \sigma \cdot \sqrt{\lambda} \cdot dw_\lambda \\
\lambda\left(t + dt\right) &= \lambda\left(t\right) + a\left(b - \lambda\left(t\right)^+\right) \cdot dt + \sigma \cdot \sqrt{\lambda\left(t\right)^+} \cdot dw_\lambda \\
\lambda\left(t\right)^+ &\equiv Max\left(\lambda\left(t\right), 0\right) \\
d\xi &= a_\xi \left(b_\xi - \xi\right) \cdot dt + \sigma_\xi \cdot \sqrt{\xi} \cdot dw_\xi \\
\xi\left(t + dt\right) &= \xi\left(t\right) + a_\xi \left(b_\xi - \xi\left(t\right)^+\right) \cdot dt + \sigma_\xi \cdot \sqrt{\xi\left(t\right)^+} \cdot dw_\xi \\
\xi\left(t\right)^+ &\equiv Max\left(\xi\left(t\right), 0\right)
\end{aligned}
$$

Appendix C

Denote with $P(\lambda, \xi, t, T_j)$ the probability, calculated at the present time t, that the debtor does not default in the time interval $[t, T_j]$, with $T_j > t$, given that the debtor has not yet defaulted at time t. Thus $P(\lambda, \xi, t, T_j)$ is the probability of survival up to time T_j. We assume that, given that the debtor has not yet defaulted at T_j, the probability of default in the infinitesimally short time interval $[T_j, T_j + dt]$ is

$$(\gamma\lambda + \xi) \cdot dt$$

Notice that the event of default in any infinitesimal interval dt is independent of the event of survival/default in any other time interval. Thus, the probability of default in the time interval $[T_j, T_j + dt]$ computed at time t and given that the debtor has not yet defaulted at t, is

$$P(\lambda, \xi, t, T_j) \cdot (\gamma\lambda + \xi) \cdot dt$$

which is the probability of surviving during $[t, T_j]$ times the probability of default during $[T_j, T_j + dt]$.

It also follows that $P(\lambda, \xi, t, T_j + dt)$ is equal to the probability of surviving up to T_j multiplied by the probability of surviving during $[T_j, T_j + dt]$, i.e.

$$P(\lambda, \xi, t, T_j + dt) = P(\lambda, \xi, t, T_j) \cdot (1 - (\gamma\lambda + \xi)\, dt).$$

We can re-write the last equation as

$$\frac{P(\lambda, \xi, t, T_j + dt) - P(\lambda, \xi, t, T_j)}{dt} = -(\gamma\lambda + \xi)\, P(\lambda, \xi, t, T_j)$$

and we can take the limit of this equation for $dt \to 0$ to obtain

$$\lim_{dt \to 0} \frac{P(\lambda, \xi, t, T_j + dt) - P(\lambda, \xi, t, T_j)}{dt} = \frac{\partial P(\lambda, \xi, t, T_j)}{\partial T_j} = -(\gamma\lambda + \xi)\, P(\lambda, \xi, t, T_j)$$

subject to $P(t, t) = 1$, since we assume that the debtor has not yet defaulted at t. $P(t, t) = 1$ because survival is certain at time t. Now the solution to

$$\frac{\partial P(\lambda, \xi, t, T_j)}{\partial T_j} = -(\gamma\lambda + \xi)\, P(\lambda, \xi, t, T_j) \quad s.t. \quad P(\lambda, \xi, t, t) = 1$$

is

$$P(\lambda, \xi, t, T_j) = e^{-(\gamma\lambda + \xi)(T_j - t)}.$$

When both λ and ξ are stochastic processes

$$P\left(\lambda,\xi,t,T_j\right)=e^{-\int\limits_t^{T_j}(\gamma\lambda_s+\xi_s)ds}.$$

Following this logic,

$$\begin{aligned}P\left(\lambda,\xi,t,T_{j-1}\right)-P\left(\lambda,\xi,t,T_j\right)&=P\left(\lambda,\xi,t,T_{j-1}\right)-P\left(\lambda,\xi,t,T_{j-1}\right)\left(1-\left(\gamma\lambda+\xi\right)dt\right)\\&=P\left(\lambda,\xi,t,T_{j-1}\right)\left(1-1+\left(\gamma\lambda+\xi\right)dt\right)\\&=P\left(\lambda,\xi,t,T_{j-1}\right)\left(\gamma\lambda+\xi\right)dt\end{aligned}$$

$$P\left(\lambda,\xi,t,T_{j-1}\right)\left(\gamma\lambda+\xi\right)=-\frac{\partial P\left(\lambda,\xi,t,T_{j-1}\right)}{\partial T_{j-1}}$$

and

$$P\left(\lambda,\xi,t,T_j\right)\left(\gamma\lambda+\xi\right)=-\frac{\partial P\left(\lambda,\xi,t,T_j\right)}{\partial T_j}$$

Appendix D

For the case we make the Recovery Face value assumption:

$$\frac{\partial V}{\partial t}+\gamma\frac{\partial^2 V}{\partial\lambda\partial r}\rho s\left(r,t\right)s\left(\lambda,t\right)+\frac{\partial^2 V}{\partial\xi\partial r}\rho s\left(r,t\right)s\left(\xi,t\right)+ \tag{19}$$

$$+\gamma\frac{\partial^2 V}{\partial\lambda^2}\frac{s\left(\lambda,t\right)^2}{2}+\frac{\partial^2 V}{\partial\xi^2}\frac{s\left(\xi,t\right)^2}{2}+\frac{\partial^2 V}{\partial r^2}\frac{s\left(r,t\right)^2}{2} \tag{20}$$

$$+\frac{\partial V}{\partial\lambda}m^*\left(\lambda,t\right)+\frac{\partial V}{\partial r}m^*\left(r,t\right)-\left(r+\lambda\right)V+\lambda R=0$$

$$\lim_{r\to\infty}V\left(\lambda,r,t\right)\to 0 \tag{21}$$

$$\lim_{r\to\infty}V\left(\lambda,\xi,r,t,T_j\right)\to 0 \tag{22}$$

$$\lim_{\lambda\to\infty}V\left(\lambda,\xi,r,t,T_j\right)\to R$$

$$\lim_{\lambda\to 0}V\left(\lambda,\xi,r,t,T_j\right)\to B\left(t,T\right)$$

$$V\left(\lambda,\xi,r,T_j,T_j\right)=1.$$

The conditions $\lim_{r\to\infty}VV\left(\lambda,\xi,r,t,T_j\right)\to 0$ and $V\left(\lambda,\xi,r,T_j,T_j\right)=1$ are the same as for default-free bonds. The lower boundary condition for r is omitted since it depends on the specific process of r. The condition $\lim_{\lambda\to\infty}VV\left(\lambda,\xi,r,t,T_j\right)\to R$ states that, as $\lambda\to\infty$ immediate default will take place, so that the bond value approaches the recovery value R. The condition $\lim_{\lambda\to 0}V\left(\lambda,r,T\right)\to B\left(t,T\right)$ states that, as $\lambda\to 0$ the bond value approaches the value of a default-free zero coupon bond. Knowing the pricing PDE 19, Feynman-Kac theorem tells us that, if $R=0$,

$$V\left(\lambda,\xi,r,t,T_j\right)=E_t^*\left[e^{-\int_t^{T_j}\left(r_\tau+\gamma\lambda_\tau+\xi_\tau\right)\cdot d\tau}\right]$$

where $E_t^*\left[..\right]$ denotes the time t conditional expectation in the risk-neutral world. This equation suggests that, assuming $R=0$, default risk causes the discount rate to increase from r to $r+\lambda$. If $\rho=0$, r , λ and ξ are independent and

$$\begin{aligned}P\left(\lambda,\xi,t,T_{j-1}\right)-P\left(\lambda,\xi,t,T_j\right)&=P\left(\lambda,\xi,t,T_{j-1}\right)-P\left(\lambda,\xi,t,T_{j-1}\right)\left(1-\left(\gamma\lambda+\xi\right)dt\right)\\&=P\left(\lambda,\xi,t,T_{j-1}\right)\left(1-1+\left(\gamma\lambda+\xi\right)dt\right)\\&=P\left(\lambda,\xi,t,T_{j-1}\right)\left(\gamma\lambda+\xi\right)dt\end{aligned}$$

$$P\left(\lambda,\xi,t,T_{j-1}\right)\left(\gamma\lambda+\xi\right)=-\frac{\partial P\left(\lambda,\xi,t,T_{j-1}\right)}{\partial T_{j-1}}$$

$$
\begin{aligned}
\left(P\left(\lambda,\xi,t,T_{j-1}\right)-P\left(\lambda,\xi,t,T_{j}\right)\right) &= -\frac{\partial P\left(\lambda,\xi,t,T_{j-1}\right)}{\partial T_{j-1}} \\
&= -\frac{\partial\left(e^{A(t,T_{j-1},a,b,\sigma)-\gamma\lambda\cdot B(t,T_{j-1},a,b,\sigma)}e^{A\left(t,T_{j-1},a_{\xi},b_{\xi},\sigma_{\xi}\right)-\xi\cdot B\left(t,T_{j-1},a_{\xi},b_{\xi},\sigma_{\xi}\right)}\right)}{\partial T_{j-1}} \\
&= \left(-\frac{\partial A\left(t,T_{j-1},a,b,\sigma\right)}{\partial T_{j-1}}+\gamma\lambda\frac{\partial B\left(t,T_{j-1},a,b,\sigma\right)}{\partial T_{j-1}}\right)P\left(\lambda,\xi,t,T_{j-1}\right) \\
&+\left(-\frac{\partial A\left(t,T_{j-1},a_{\xi},b_{\xi},\sigma_{\xi}\right)}{\partial T_{j-1}}+\xi\frac{\partial B\left(t,T_{j-1},a_{\xi},b_{\xi},\sigma_{\xi}\right)}{\partial T_{j-1}}\right)P\left(\lambda,\xi,t,T_{j-1}\right)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial B\left(t,T_{j-1},a,b,\sigma\right)}{\partial T_{j-1}} &= -\frac{1}{2}\sigma^{2}B^{2}\left(t,T_{j-1},a,b,\sigma\right)-aB\left(t,T_{j-1},a,b,\sigma\right)+1 \\
\frac{\partial A\left(t,T_{j-1},a,b,\sigma\right)}{\partial T_{j-1}} &= -abB\left(t,T_{j-1},a,b,\sigma\right)
\end{aligned}
$$

Considering annual quotations for bonds with maturity j :

$$
\overline{s}_{j,t}=(1-R)\cdot\frac{\sum_{k=1}^{j}D\left(r,t,t+k\right)\cdot\left(P\left(\lambda,\xi,t,t+(k-1)\right)-P\left(\lambda,\xi,t,t+k\right)\right)}{\sum_{k=1}^{j}V\left(\lambda,\xi,r,t,t+k\right)}
$$

it is the risk-neutral probability of default for infinitesimal period $[T_{k-1},T_{k}]$ given that no default has occurred at T_{k-1} where $T_{k}\equiv t+k$

$$
\overline{s}_{j,t}=(1-R)\cdot\frac{\sum_{k=1}^{j}D\left(r,t,t+k\right)\cdot\left(-\frac{\partial P(\lambda,\xi,t,t+(k-1))}{\partial(t+(k-1))}\right)}{\sum_{k=1}^{j}V\left(\lambda,\xi,r,t,t+k\right)}
$$

since:

$$
E_{t}^{*}\left[\lambda_{t_{j}}e^{-\int_{t}^{T_{j}}\lambda_{\tau}\cdot d\tau}\right]=E_{t}^{*}\left[\lambda_{t_{j}}\cdot P\left(\lambda,\xi,t,T_{j}\right)\right]=\left(-\frac{\partial P\left(\lambda,\xi,t,T_{j-1}\right)}{\partial T_{j-1}}\right)\simeq\left(P\left(\lambda,\xi,t,T_{j-1}\right)-P\left(\lambda,\xi,t,T_{j}\right)\right)
$$

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Table 1: Summary statistics for one-year sovereign CDS spreads

	mean	sd	min	p50	max
Austria	60.50	40.95	16.62	46.30	253.40
Belgium	55.98	30.53	13.75	50.87	136.35
Finland	14.19	11.77	3.86	7.64	56.75
France	31.24	14.53	7.29	29.92	63.61
Germany	20.16	10.60	5.55	19.97	61.63
Greece	446.13	373.20	64.15	222.89	1299.62
Ireland	215.14	127.69	91.43	164.87	654.06
Italy	100.32	48.06	20.44	103.20	241.41
Netherlands	29.98	22.32	7.78	24.39	102.31
Portugal	166.13	136.89	28.75	90.03	494.17
Spain	121.82	78.64	27.45	89.47	316.70

Note: computed from 522 daily observations for 2009–2010.

Table 2: Summary statistics for five-year sovereign CDS spreads

	mean	sd	min	p50	max
Austria	93.76	40.68	48.92	83.07	273.00
Belgium	87.53	44.65	30.78	71.81	219.77
Finland	33.62	14.87	16.01	29.52	93.92
France	55.66	24.18	21.01	54.50	108.25
Germany	38.90	14.51	19.01	37.46	91.85
Greece	426.29	308.09	100.26	281.33	1125.81
Ireland	249.93	128.27	114.65	211.36	620.19
Italy	136.86	49.88	56.78	128.75	268.27
Netherlands	50.33	23.70	25.00	45.00	131.00
Portugal	185.90	135.40	44.57	130.67	542.17
Spain	148.82	75.48	54.90	124.23	364.20

Note: computed from 522 daily observations for 2009–2010.

Table 3: Summary statistics for one-year sovereign CDS spreads vs. Germany

	mean	sd	min	p50	max
Austria	40.33	33.70	3.98	31.79	200.93
Belgium	35.82	26.35	5.30	28.97	122.20
Finland	-5.97	7.72	-24.24	-3.75	6.25
France	11.08	13.08	-10.83	5.09	49.97
Greece	425.97	372.76	55.15	175.80	1273.11
Ireland	194.98	126.67	71.30	141.01	629.15
Italy	80.16	43.36	11.45	73.96	214.61
Netherlands	9.82	14.61	-13.24	3.56	58.10
Portugal	145.97	136.43	18.22	47.40	462.02
Spain	101.66	77.61	21.73	58.42	290.58

Note: computed from 522 daily observations for 2009–2010.

Table 4: Summary statistics for five-year sovereign CDS spreads vs. Germany

	mean	sd	min	p50	max
Austria	54.86	29.49	17.31	46.63	182.75
Belgium	48.63	35.93	9.73	29.86	161.10
Finland	-5.29	7.30	-27.34	-3.64	15.60
France	16.76	16.35	-0.23	8.96	52.15
Greece	387.39	304.45	76.88	252.83	1081.95
Ireland	211.02	121.84	88.75	168.84	561.55
Italy	97.96	41.25	33.39	89.79	212.55
Netherlands	11.43	12.33	-7.36	7.64	66.30
Portugal	147.00	131.35	20.92	77.50	486.61
Spain	109.92	70.39	31.52	79.36	308.48

Note: computed from 522 daily observations for 2009–2010.

Table 5: Correlations of changes in one-year sovereign CDS spreads

	dAUT	dBEL	dDEU	dESP	dFIN	dFRA	dGRC	dIRL	dITA	dNLD	dPRT
dAUT	1.00										
dBEL	0.46	1.00									
dDEU	0.26	0.42	1.00								
dESP	0.31	0.56	0.40	1.00							
dFIN	0.73	0.41	0.36	0.25	1.00						
dFRA	0.50	0.57	0.56	0.56	0.51	1.00					
dGRC	0.16	0.31	0.27	0.46	0.14	0.43	1.00				
dIRL	0.32	0.46	0.49	0.51	0.31	0.46	0.42	1.00			
dITA	0.50	0.56	0.37	0.83	0.39	0.61	0.50	0.52	1.00		
dNLD	0.50	0.76	0.40	0.36	0.51	0.49	0.20	0.41	0.41	1.00	
dPRT	0.22	0.42	0.34	0.68	0.17	0.49	0.61	0.64	0.67	0.25	1.00

Note: computed from 521 daily observations for 2009–2010.

Table 6: Correlations of changes in five-year sovereign CDS spreads

	dAUT	dBEL	dDEU	dESP	dFIN	dFRA	dGRC	dIRL	dITA	dNLD	dPRT
dAUT	1.00										
dBEL	0.67	1.00									
dDEU	0.70	0.67	1.00								
dESP	0.51	0.71	0.65	1.00							
dFIN	0.73	0.57	0.67	0.47	1.00						
dFRA	0.68	0.74	0.80	0.71	0.67	1.00					
dGRC	0.36	0.49	0.46	0.69	0.32	0.58	1.00				
dIRL	0.58	0.64	0.59	0.77	0.53	0.64	0.60	1.00			
dITA	0.57	0.73	0.66	0.89	0.52	0.73	0.64	0.74	1.00		
dNLD	0.74	0.73	0.65	0.55	0.67	0.64	0.37	0.60	0.60	1.00	
dPRT	0.42	0.62	0.56	0.86	0.38	0.66	0.78	0.76	0.80	0.46	1.00

Note: computed from 521 daily observations for 2009–2010.

Table 7: Summary statistics from SUR estimation over full sample

Cty	N	np	RMSE	R^2	χ^2	p-val
AUT	5210	29	5.925394	0.9797	293684.11	0.0000
BEL	5210	29	4.824921	0.9876	475299.33	0.0000
DEU	5210	29	1.959797	0.9846	376326.75	0.0000
ESP	5210	29	9.769852	0.9834	373549.23	0.0000
FIN	5210	29	1.774344	0.9872	438731.95	0.0000
FRA	5210	29	2.637656	0.9887	508030.68	0.0000
GRC	5210	29	37.78125	0.9854	365146.21	0.0000
IRL	5210	29	14.40218	0.9865	428535.26	0.0000
ITA	5210	29	8.074119	0.9746	246019.63	0.0000
NLD	5210	29	2.787547	0.9871	465464.49	0.0000
PRT	5210	29	15.7709	0.9857	404045.18	0.0000

Note: N is the number of observations per equation, while np is the number of parameters per equation. RMSE is the root mean square error of the equation, while R^2 is the equation's R-squared value. The χ^2 statistic is a test that all slope coefficients are jointly zero, with associated p-val.

Table 8: Effects of German $cumpdf_{t-1}$ on troubled borrowers' CDS spreads

Borrower	Coefficient	Std. Err.	p-val
Greece	-5.2986	1.846	0.004
Ireland	6.3368	0.692	0.000
Italy	1.7634	0.389	0.000
Portugal	0.5656	0.761	0.457
Spain	-0.3204	0.470	0.496

Note: Full-sample SUR coefficients and standard errors of $cumpdfDEU_{t-1}$ on borrower's CDS spreads, with p-val of test for coefficient being significantly different from zero.

Table 9: Ex ante forecast statistics for 02sep2009–02oct2009

	RMSE	MAE	MAPE	Theil U
1yr DEU	1.392	1.265	12.738	0.127
1yr ESP	4.592	4.009	9.135	0.099
1yr GRC	12.624	9.550	11.659	0.139
1yr IRL	13.383	11.346	10.419	0.117
1yr ITA	6.114	4.110	14.718	0.162
1yr PRT	3.516	2.885	8.116	0.098
3yr DEU	1.585	1.326	10.245	0.109
3yr ESP	3.673	2.942	5.944	0.070
3yr GRC	8.819	6.262	6.635	0.088
3yr IRL	11.430	9.062	7.051	0.084
3yr ITA	4.647	3.212	8.080	0.099
3yr PRT	2.521	1.850	4.762	0.064
5yr DEU	1.332	1.147	6.888	0.077
5yr ESP	2.958	2.384	4.156	0.050
5yr GRC	6.405	4.716	4.382	0.059
5yr IRL	8.902	7.160	5.177	0.062
5yr ITA	2.635	2.284	4.123	0.046
5yr PRT	2.061	1.553	3.614	0.047

Table 10: Ex ante forecast statistics for 03may2010–02jun2010

	RMSE	MAE	MAPE	Theil U
1yr DEU	1.751	1.407	5.576	0.071
1yr ESP	25.943	21.504	12.282	0.144
1yr GRC	130.089	89.547	10.528	0.146
1yr IRL	26.551	21.153	10.227	0.125
1yr ITA	20.802	15.757	10.391	0.138
1yr PRT	55.691	40.004	12.279	0.161
3yr DEU	1.919	1.498	4.906	0.065
3yr ESP	26.037	20.699	10.997	0.132
3yr GRC	113.402	73.370	9.229	0.136
3yr IRL	26.116	19.586	8.961	0.114
3yr ITA	21.337	15.855	9.713	0.130
3yr PRT	51.481	34.712	11.135	0.155
5yr DEU	2.363	1.813	4.930	0.066
5yr ESP	25.968	20.433	10.399	0.126
5yr GRC	101.315	68.868	8.932	0.127
5yr IRL	24.727	18.169	8.387	0.108
5yr ITA	22.415	16.528	9.514	0.128
5yr PRT	49.948	33.363	10.826	0.152

Table 11: Ex ante forecast statistics for 29nov2010–29dec2010

	RMSE	MAE	MAPE	Theil U
1yr DEU	2.407	1.539	10.461	0.182
1yr ESP	19.957	18.684	8.489	0.091
1yr GRC	78.663	75.734	7.719	0.080
1yr IRL	46.129	42.167	8.403	0.093
1yr ITA	11.177	9.698	7.771	0.090
1yr PRT	29.704	28.059	8.419	0.090
3yr DEU	1.860	1.460	6.286	0.084
3yr ESP	16.675	14.257	4.940	0.058
3yr GRC	39.055	35.584	3.343	0.037
3yr IRL	32.344	24.912	4.509	0.060
3yr ITA	9.953	8.124	5.067	0.063
3yr PRT	23.624	21.039	4.780	0.054
5yr DEU	1.820	1.629	4.739	0.053
5yr ESP	15.390	12.759	4.101	0.049
5yr GRC	23.247	20.632	2.051	0.023
5yr IRL	26.813	18.724	3.206	0.047
5yr ITA	11.014	8.694	4.459	0.057
5yr PRT	20.445	17.587	3.705	0.043

Figure 1: Correlations of changes in risky borrowers' CDS spreads

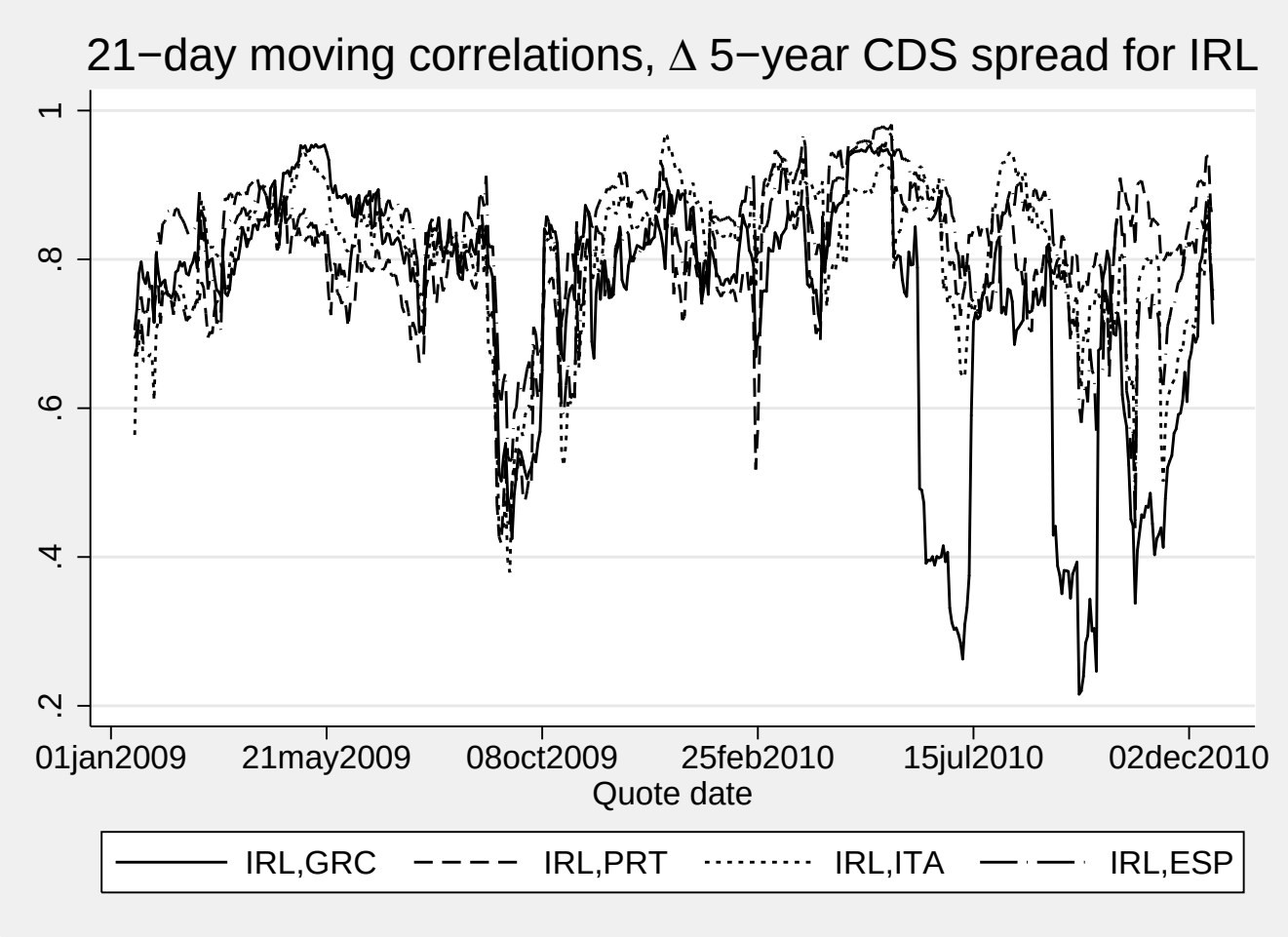

Figure 2: Correlations of changes in risky borrowers' CDS spreads

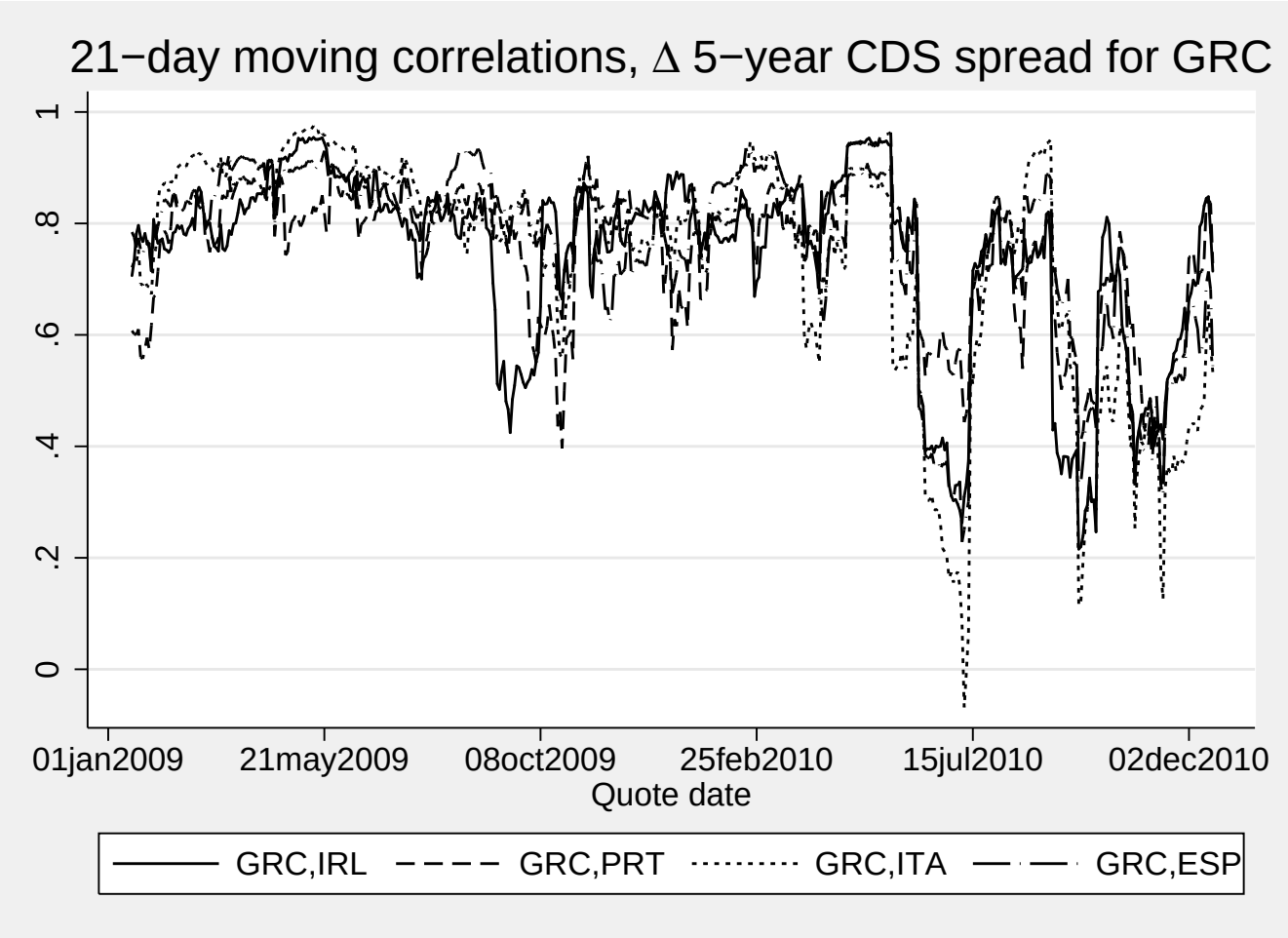

Figure 3: Moving-window estimates of CDS spread volatility

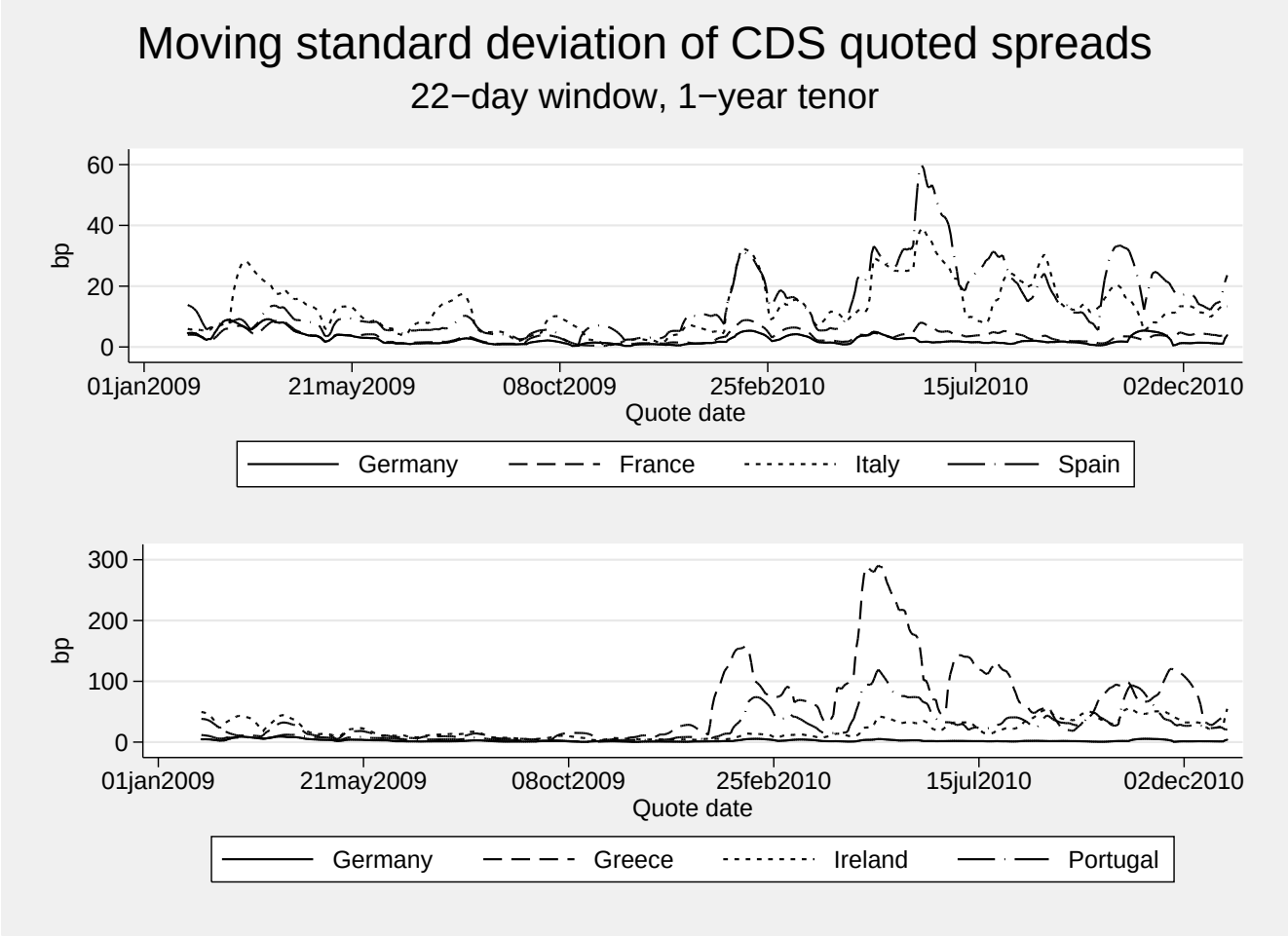

Figure 4: Moving-window estimates of CDS spread volatility

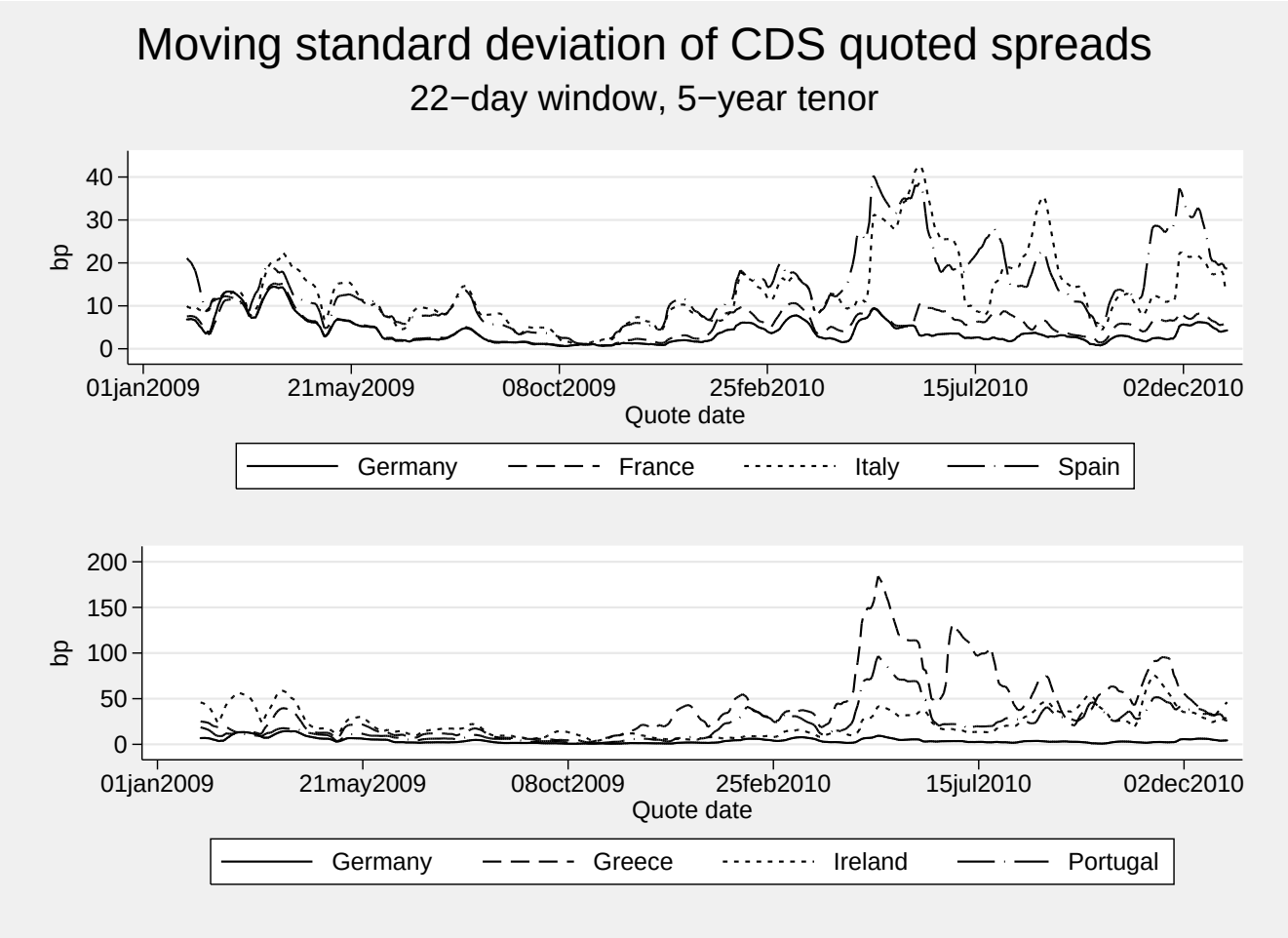

Figure 5: Cross-sectional estimates of CDS spread volatility

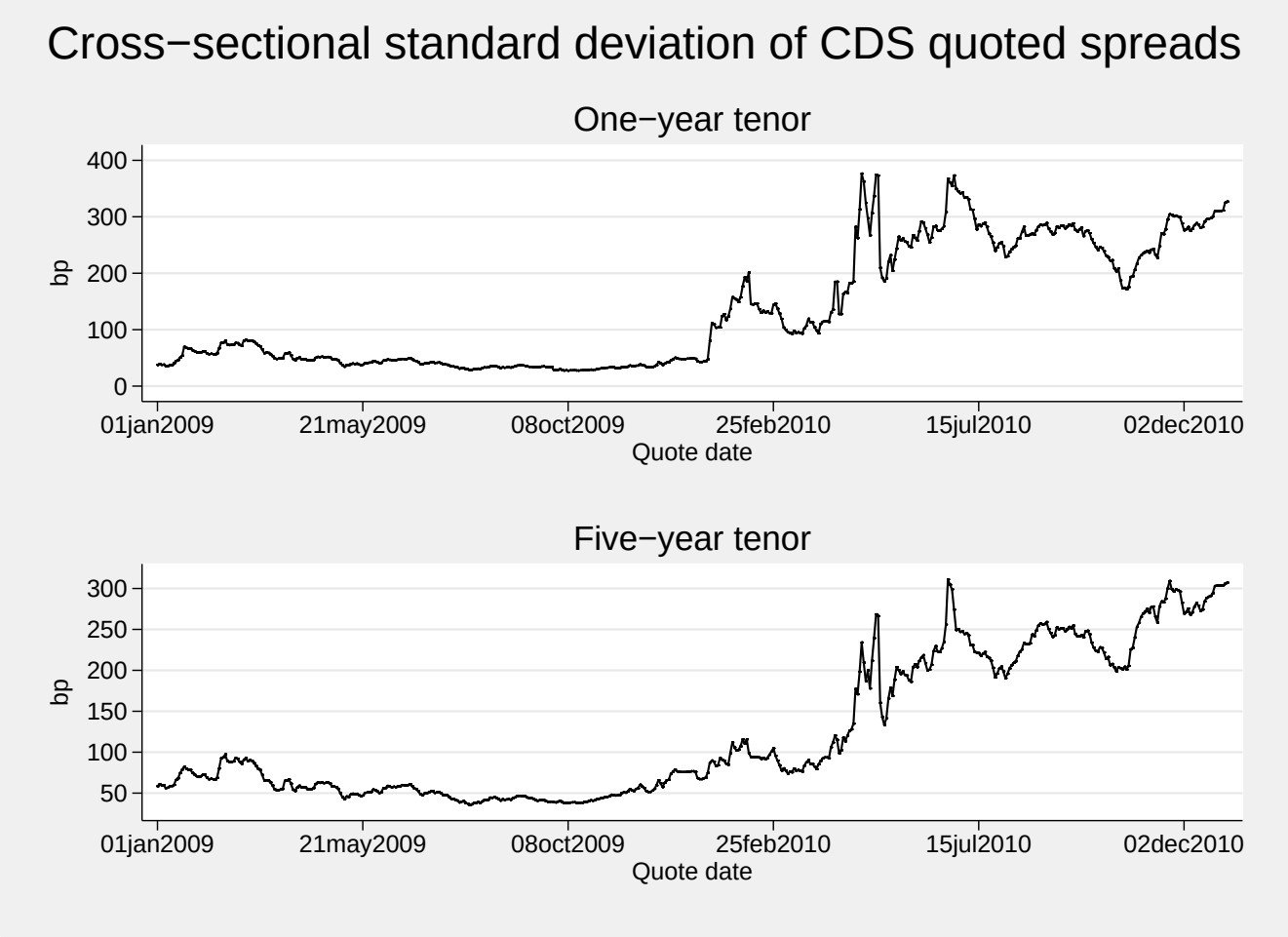
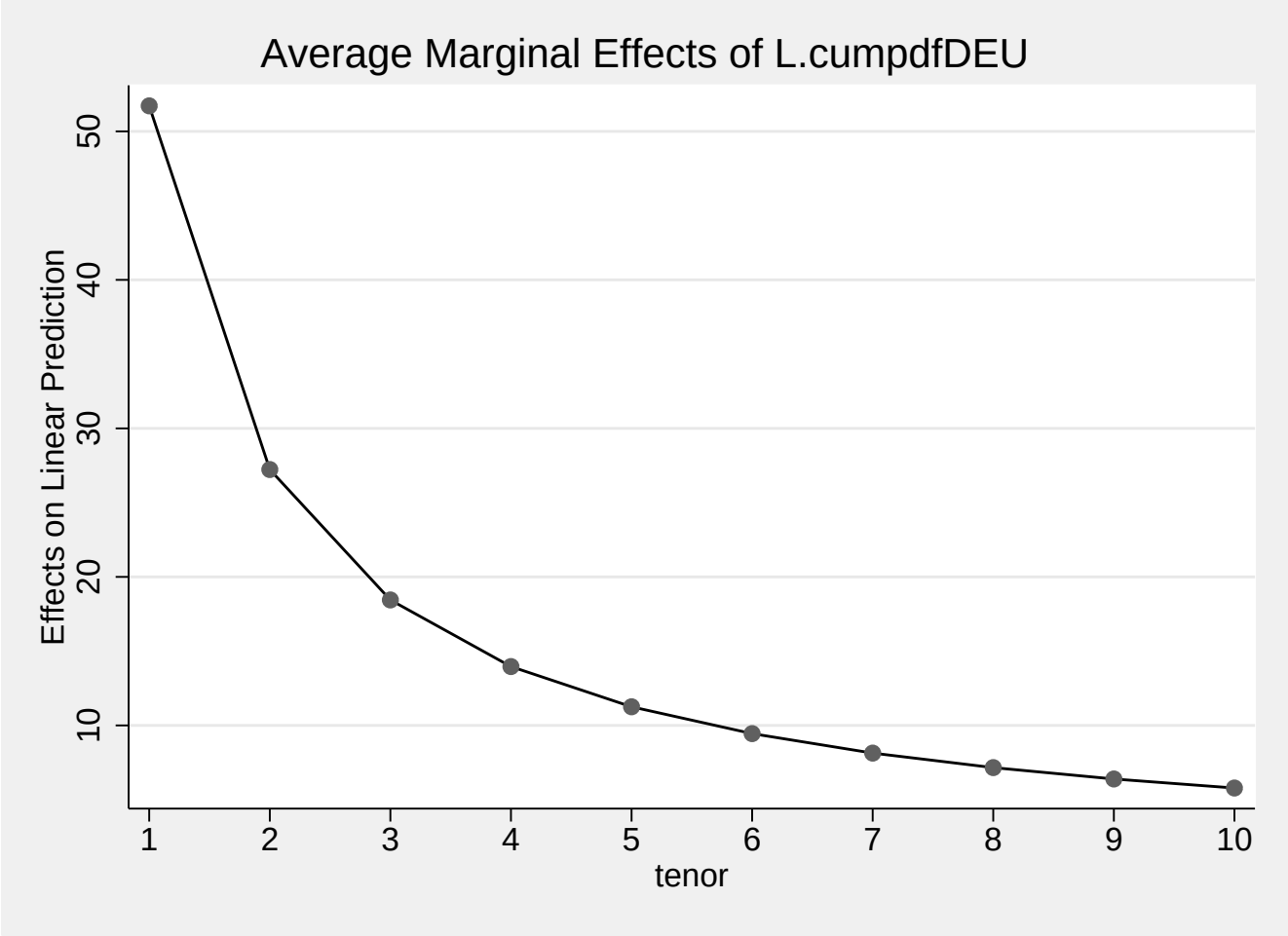
Figure 6: Effects of cumpdf on German CDS spreads by tenor