Deriving the Taylor Principle when the Central Bank Supplies Money

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Abstract

The paper presents a human-capital-based endogenous growth, cash-in-advance economy with endogenous velocity where exchange credit is produced in a decentralized banking sector, and money supplied stochastically by the central bank. From this it derives an exact functional form for a general equilibrium 'Taylor rule'. The inflation coefficient is always greater than one when the velocity of money exceeds one; velocity growth enters the equilibrium condition as a separate variable. The paper then successfully estimates the magnitude of the coefficient on inflation from 1000 samples of Monte Carlo simulated data. This shows it is spurious to conclude that the central bank has a reaction function with a 'tough' inflation policy in a 'Taylor principle' sense, since instead it is only meeting fiscal needs through the inflation tax. The paper also estimates several deliberately misspecified models to show how an inflation coefficient of less than one can result from model misspecification. An inflation coefficient greater than one holds theoretically along the balanced growth path equilibrium, making it a sharply robust principle based on the economy's underlying structural parameters as consistent with the Lucas (1976) research agenda for policy rule formation.

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1 Introduction

Interest rate rules are widely considered as monetary policy 'reaction functions' that represent how the central bank adjusts a short-term nominal interest rate in response to the state of the economy. The magnitude of the reaction function coefficients are interpreted to reflect a policymaker's attitude towards variation in key macroeconomic variables such as inflation and the output gap. It has been suggested that policymakers ought to adhere to the 'Taylor principle', whereby inflation above target is met by a more-than-proportional increase in the short-term nominal interest rate and hence an increase in the real interest rate. Such an interest rate rule forms one of the three core equations of the prominent New Keynesian modelling framework (e.g. Clarida et al., 1999; Woodford, 2003); Clarida et al. (2000) conclude that the Taylor principle holds for a 'Volcker-Greenspan' sample of U.S. data but that it is violated for a 'pre-Volcker' sample.

In contrast, a historical strand of literature going back to Poole (1970) and updated for example by Alvarez et al. (2001) and Chowdhury and Schabert (2008) who consider interest rate rules and money supply rules as two ways of implementing the same monetary policy. This paper perhaps most closely follows Alvarez et al. (2001) by deriving the equilibrium nominal interest rate in a 'rule' form within a general equilibrium economy in which the central bank conducts policy by stochastically supplying money. Instead of an exogenous fraction of agents being able to use bonds as in Alvarez et al., here the consumer purchases goods with an endogenous fraction of bank-supplied intratemporal credit that avoids the inflation tax on exchange. This cash-in-advance monetary economy also is extended to include endogenous growth, along with endogenous velocity, as in Benk et al. (2010). The resulting equilibrium nominal interest rate condition 'nests' the standard Taylor rule within a more general forward-looking setting that endogenously includes traditional monetary elements, such as the (exogenous) velocity in Alvarez et al., and the money demand in McCallum and Nelson (1999).

The endogenous growth aspect make the equilibrium 'Taylor condition' contain 'target' terms such as the inflation rate target or 'potential' output level endogenous structural coefficients that are determined by the economy's stationary balanced growth path (BGP) equilibrium. This in essence fulfills Lucas's (1976) goal of having policy rule coefficients that depend explicitly upon the economy's underlying utility and technology coefficients. Aesthic as such a formulation of the Taylor condition may be, Lucas's research agenda provides a solid result: a theoretical derivation of the 'Taylor principle' coefficient on the inflation term that shows it always exceeds or equals one. The 'principle' holds for given any non-Friedman (1969) optimum BGP money supply growth rate; it equals one only at the Friedman optimum, and never falls below one. Similarly, the inflation coefficient always exceeds one when the endogenous velocity exceeds one since the cash-in-advance velocity rises above one for any non-Friedman optimal rate of money supply growth. Not only can the inflation coefficient be written in terms of velocity, whereby the coefficient rises with the BGP velocity level, but an additional central result comes in terms of velocity.

The expected velocity growth rate itself enters the Taylor condition as a separate additional term, relative to standard Taylor rules. Omitting this term can cause misspecification bias in a Taylor rule estimation within the economy.

Given the derived Taylor condition, the paper then estimates it using one thousand samples of artificial data as simulated from the baseline model, and using OLS, two-stage least squares and GMM estimators. The results verify the theoretical Taylor condition along key dimensions, in particular with the coefficient on inflation greater than one and close to the theoretical value of the coefficient. Satisfying the 'Taylor principle' in this fashion, robustness tests explore what happens when a related but different Taylor condition is instead estimated. This involves two ad hoc deliberately misspecificed equations relative to the theory: the first changes just one of the variables in the Taylor condition while the second posits a standard Taylor rule that involves multiple misspecification errors. Using the same artificial data, estimation of the two misspecified models results in the coefficient on inflation falling below one, causing the 'Taylor principle' to fail. In actual data estimation, this finding would typically be interpreted as the central bank being 'passive' or 'weak' towards inflation. Here, the paper shows that such an interpretation could be spurious in that it could occur simply because of a misspecified estimating equation.[1]

The estimated 'Taylor rule' emerges even though the central bank is merely satisfying fiscal needs through the inflation tax. This implies the central point of the paper: it would be spurious within this economy to associate the Taylor condition with a 'reaction function' for the nominal interest rate since in the model the central bank just stochastically prints money. Second, failure of the so-called Taylor principle in numerous published empirical studies may be a result of model misspecification rather than behavioral changes by the bank per se. Indeed, our current preliminary extension of this work, not presented here, shows that estimation with actual US data of the Taylor rules using the additional unconventional terms implied by the theory of this paper - particularly velocity growth - can reverse the result that the coefficient on inflation falls below unity during periods of macroeconomic instability.[2]

Related work is vast but includes Taylor (1999), who alludes to the possibility that an interest rate rule can be derived from the quantity theory of money. Sørensen and Whitta-Jacobsen (2005, pp.502-505) present a derivation under the assumption of constant money growth whereby the coefficients of the 'rule' relate to elasticities of money demand rather than the preferences of policymakers. Fève and Auray (2002) and Schabert (2003) consider the link between money supply rules and interest rate rules in standard cash-in-advance models with velocity fixed at unity. Or the paper could be viewed in light of Canzoneri et al. (2007) in that it shows how the puzzle of successfully estimating the Euler equation of the interest rate can be solved by combining that equilibrium

[1] Estimation of simulated data is conducted by Fève and Auray (2002), for a standard CIA model, and Salyer and Van Gaasbeck (2007), for a 'limited participation' model. We are indebted to Warren Weber for the suggestion to follow such an approach here.

[2] Clarida et al.'s (2000) 'pre-Volcker' sample, for example, corresponds to a period of high and variable inflation.

condition with the stochastic asset pricing kernel to derive an expression for a Taylor condition that can find success in estimation.

Section 2 describes the economy, as in Benk et al. (2008,2010). Section 3 derives the model's 'Taylor condition' and Section 4 provides the baseline calibration. Section 5 describes the econometric methodology uses to estimate the data that is simulated from the baseline model; it then presents the estimation results. Section 6 derives special theoretical cases of the more general (Section 2) model to show alternative Taylor conditions, while Section 7 presents a discussion and Section 8 concludes.

2 Stochastic Endogenous Growth with Banking

The representative agent economy is as in Benk et al (2008, 2010) but with a decentralized banking sector that produces credit as in Gillman and Kejak (2011). By combining the business cycle with endogenous growth, stationary inflation lowers the output growth rate as supported empirically in Gillman et al. (2004) and Fountas et al. (2006), for example. Further, money supply shocks can cause inflation at low frequencies, as in Haug and Dewald (2011) and as supported by Sargent and Surico (2008, 2011), which can lead to output growth effects if the shocks are persistent and repeated. This allows shocks over the business cycle to cause changes in growth rates and in stationary ratios. The shocks to the goods sector productivity and the money supply growth rate are standard, while the third shock to credit sector productivity exists by virtue of the model's endogenous money velocity via the production function used extensively in the financial intermediation microeconomics literature starting with Clark (1984).

The shocks occur at the beginning of the period, are observed by the consumer before the decision making process commences, and follow a vector first-order autoregressive process. For goods sector productivity, z_t, the money supply growth rate, u_t, and bank sector productivity, v_t:

$$Z_t = \Phi_Z Z_{t-1} + \varepsilon_{Zt}, \tag{1}$$

where the shocks are $Z_t = [z_t \; u_t \; v_t]'$, the autocorrelation matrix is $\Phi_Z = diag\{\varphi_z, \varphi_u, \varphi_v\}$ and $\varphi_z, \varphi_u, \varphi_v \in (0,1)$ are autocorrelation parameters, and the shock innovations are $\varepsilon_{Zt} = [\epsilon_{zt} \; \epsilon_{ut} \; \epsilon_{vt}]' \sim N(\mathbf{0}, \mathbf{\Sigma})$. The general structure of the second-order moments is assumed to be given by the variance-covariance matrix $\mathbf{\Sigma}$. These shocks affect the economy as described below.

2.1 Consumer Problem

A representative consumer has expected lifetime utility from consumption of goods, c_t, and leisure, x_t; with $\beta \in (0,1)$ and $\theta > 0$, this is given by:

$$U = E_0 \sum_{t=0}^{\infty} \beta \frac{(c_t x_t^{\psi})^{1-\theta}}{1-\theta}. \tag{2}$$

Output of goods, y_t, and increases in human capital, are produced with physical capital and effective labor each in Cobb-Douglas fashion; the bank sector produces exchange credit using labor and deposits as inputs. Let s_{Gt} and s_{Ht} denote the fractions of physical capital that the agent uses in the goods production (G) and human capital investment (H), whereby:

$$s_{Gt} + s_{Ht} = 1. \tag{3}$$

The agent allocates a time endowment of one between leisure, x_t, labor in goods production, l_t, time spent investing in the stock of human capital, n_t, and time spent working in the bank sector, denoted by f_t:

$$l_t + n_t + f_t + x_t = 1. \tag{4}$$

Output of goods can be converted into physical capital, k_t, without cost and so is divided between consumption goods and investment, denoted by i_t, net of capital depreciation. Thus, the capital stock used for production in the next period is given by:

$$k_{t+1} = (1 - \delta_k)k_t + i_t = (1 - \delta_k)k_t + y_t - c_t. \tag{5}$$

The human capital investment is produced using capital $s_{Ht}k_t$ and effective labor $n_t h_t$:

$$H(s_{Ht}k_t, n_t h_t) = A_H(s_{Ht}k_t)^{1-\eta}(n_t h_t)^{\eta}. \tag{6}$$

And the human capital flow constraint is:

$$h_{t+1} = (1 - \delta_h)h_t + H(s_{Ht}k_t, n_t h_t). \tag{7}$$

With w_t and r_t denoting the real wage and real interest rate, the consumer receives nominal income of wages and rents, $P_t w_t \left(l_t + f_t\right) h_t$ and $P_t r_t \left(s_{Gt} + s_{Qt}\right) k_t$, a nominal transfer from the government, T_t, and dividends from the bank.

The consumer buys shares in the bank by making deposits of income at the bank. Each dollar deposited buys one share at a fixed price of one, and the consumer receives the residual profit of the bank as dividend income in proportion to the number of shares (deposits) owned. Denoting the real quantity of deposits by d_t, and the dividend per unit of deposits as R_{Qt}, the consumer receives a nominal dividend income of $P_t R_{Qt} d_t$. The consumer also pays to the bank a fee for credit services, whereby one unit of credit service is required for each unit of credit that the bank supplies the consumer for use in buying goods. With P_{Qt} denoting the nominal price of each unit of credit, and q_t the real quantity of credit that the consumer can use in exchange, the consumer pays $P_{Qt} q_t$ in credit fees.

With other expenditures on goods, of $P_t c_t$, and physical capital investment, $P_t k_{t+1} - P_t(1 - \delta_k)k_t$, and on investment in cash for purchases, of $M_{t+1} - M_t$, and in nominal bonds $B_{t+1} - B_t(1 + R_t)$, where R_t is a net interest rate, the consumer's budget constraint is:

$$
\begin{aligned}
& P_t w_t \left(l_t + f_t\right) h_t + P_t r_t s_{Gt} k_t + P_t R_{Qt} d_t + T_t \\
\geq \; & P_{Qt} q_t + P_t c_t + P_t k_{t+1} - P_t (1 - \delta_k) k_t + M_{t+1} - M_t \\
& + B_{t+1} - B_t (1 + R_t).
\end{aligned} \tag{8}
$$

The consumer can purchase the goods by using either money M_t or credit services. With the lump sum transfer of cash T_t coming from the government at the beginning of the period, and with money and credit equally usable to buy goods, the consumer's exchange technology is:

$$M_t + T_t + P_t q_t \geq P_t c_t. \tag{9}$$

Since all cash comes out of deposits at the bank, and credit purchases are paid off at the end of the period out of the same deposits, the total deposits are equal to consumption. This gives the constraint that

$$d_t = c_t. \tag{10}$$

Given k_0, h_0, and the evolution of M_t ($t \geq 0$) as given by the exogenous monetary policy in equation (18) below, the consumer maximizes utility subject to the budget, exchange and deposit constraints (8)-(10).

2.2 Banking Firm Problem

The bank produces credit that is available for exchange at the point of purchase. The bank determines the amount of such credit by maximizing its dividend profit subject to the labor and deposit costs of producing the credit. The production of credit uses a constant returns to scale technology with effective labor and deposited funds as inputs. In particular, with $A_Q > 0$ and $\gamma \in (0, 1)$,

$$q_t = A_F e^{v_t} \left(f_t h_t\right)^{\gamma} d_t^{1-\gamma}, \tag{11}$$

where $A_F e^{v_t}$ is the stochastic factor productivity.

Subject to the production function in equation (11), the bank maximizes profit Π_{Ft} with respect to the labor f_t and deposits d_t:

$$\Pi_{Ft} = P_{Ft} q_t - P_t w_t f_t h_t - P_t R_{Ft} d_t. \tag{12}$$

Equilibrium implies that

$$\left(\frac{P_{Ft}}{P_t}\right) \gamma A_F e^{v_t} \left(\frac{f_t h_t}{d_t}\right)^{\gamma - 1} = w_t; \tag{13}$$

$$\left(\frac{P_{Ft}}{P_t}\right) (1 - \gamma) A_F e^{v_t} \left(\frac{f_t h_t}{d_t}\right)^{\gamma} = R_{Ft}. \tag{14}$$

These indicate that the marginal cost of credit, $\left(\frac{P_{Ft}}{P_t}\right)$, is equal to the marginal factor price divided by the marginal factor product, or $\frac{w_t}{\gamma A_F e^{v_t} \left(\frac{f_t h_t}{d_t}\right)^{\gamma - 1}}$, and

that the zero profit dividend yield paid on deposits is equal to the fraction of the marginal cost given by $\left(\frac{P_{Ft}}{P_t}\right)(1-\gamma)\left(\frac{q_t}{d_t}\right)$.

2.3 Goods Producer Problem

The firm maximizes profit given by $y_t - w_t l_t h_t - r_t s_{Gt} k_t$, subject to a standard Cobb-Douglas production function in effective labor and capital:

$$y_t = A_G e^{z_t}(s_{Gt}k_t)^{1-\alpha}(l_t h_t)^{\alpha}. \tag{15}$$

The first order conditions for the firm's problem yield the following expressions for the wage rate and the rental rate of capital:

$$w_t = \alpha A_G e^{z_t}\left(\frac{s_{Gt}k_t}{l_t h_t}\right)^{1-\alpha}, \tag{16}$$

$$r_t = (1-\alpha)A_G e^{z_t}\left(\frac{s_{Gt}k_t}{l_t h_t}\right)^{-\alpha}. \tag{17}$$

2.4 Government Money Supply

It is assumed that the government policy includes sequences of nominal transfers which satisfy:

$$T_t = \Theta_t M_t = (\Theta^* + e^{u_t} - 1)M_t, \qquad \Theta_t = [M_t - M_{t-1}]/M_{t-1}. \tag{18}$$

where Θ_t is the growth rate of money and Θ^* is the stationary gross growth rate of money.

2.5 Definition of Competitive Equilibrium

The representative agent's optimization problem can be written recursively as:

$$V(s) = \max_{c,x,l,n,f,s_G,q,d,k',h',M'} \{u(c,x) + \beta EV(s')\} \tag{19}$$

subject to the conditions (3) to (10), where the state of the economy is denoted by $s = (k, h, M, B; z, u, v)$ and a prime (') indicates the next-period values. A competitive equilibrium consists of a set of policy functions $c(s)$, $x(s)$, $l(s)$, $n(s)$, $f(s)$, $s_G(s)$, $q(s)$, $d(s)$, $k'(s)$, $h'(s)$, $M'(s)$, $B'(s)$ pricing functions $P(s)$, $w(s)$, $r(s)$, $R_F(s)$, $P_F(s)$ and a value function $V(s)$, such that:

(i) the consumer maximize utility, given the pricing functions and the policy functions, so that $V(s)$ solves the functional equation (19);

(ii) the goods producer maximizes profit similarly, with the resulting functions for w and r being given by equations (16) and (17);

(iii) the bank firm maximizes profit similarly in equation (12) subject to the technology of equation (11)

(iv) the goods, money and credit markets clear, in equations (8) and (15), and in (9), (18), and (11).

3 General Equilibrium Taylor Condition

The 'Taylor condition' is now derived as an equilibrium condition of the Benk et al. (2010) model described in the previous section. Beginning from the first-order conditions of the model, we obtain:

$$1 = \beta E_t \left\{ \frac{c_{t+1}^{-\theta} x_{t+1}^{\psi(1-\theta)}}{c_t^{-\theta} x_t^{\psi(1-\theta)}} \frac{\tilde{R}_t}{\tilde{R}_{t+1}} \frac{R_{t+1}}{\pi_{t+1}} \right\}, \tag{20}$$

where R and π are gross rates of nominal interest and inflation, respectively, and where $\tilde{R}_t$ is a 'weighted average cost of exchange' defined as:

$$\begin{aligned} \tilde{R}_t &\equiv R_t - (1-\gamma)\left(1 - \frac{m_t}{c_t}\right)(R_t - 1); \\ (\tilde{R}_t - 1) &= \frac{m_t}{c_t}(R_t - 1) + \gamma\left(1 - \frac{m_t}{c_t}\right)(R_t - 1). \end{aligned}$$

Where, $\frac{m_t}{c_t}$ represents consumption normalized money demand (the inverse of the consumption velocity of money). In effect, equation (20) augments a standard consumption Euler equation with the growth rate of this average cost of exchange. If all transactions are conducted using money ($m_t/c_t = 1$) then equation (20) reverts back to the familiar consumption Euler equation which would feature as an equilibrium condition of a standard CIA model without a money alternative.[3]

For any variable z_t, define $\widehat{z}_t \equiv \ln z_t - \ln z$, where the absence of a time subscript denotes a BGP stationary value, and define $\widehat{g}_{z,t+1} \equiv \ln z_{t+1} - \ln z_t$, which approximates the growth rate of z at time $t+1$ (for sufficiently small z). Consider a log-linear approximation of (20) evaluated around the BGP:

$$0 = E_t \left\{ \theta \widehat{g}_{c,t+1} - \psi(1-\theta)\widehat{g}_{x,t+1} + \widehat{g}_{\tilde{R},t+1} - \widehat{R}_{t+1} + \widehat{\pi}_{t+1} \right\}.$$

Rearranging this in terms of $\widehat{R}_t$ gives the Taylor condition expressed in log-deviations from the BGP equilibrium:

$$\begin{aligned} \widehat{R}_t &= E_t \{ \Omega \widehat{\pi}_{t+1} + \Omega\theta \widehat{g}_{c,t+1} - \Omega\psi(1-\theta)\widehat{g}_{x,t+1} \\ &\quad + \frac{(1-\gamma)\left(1-\frac{m}{c}\right)}{R\left[1-(1-\gamma)\left(1-\frac{m}{c}\right)\right]} \left[(R-1)\frac{\frac{m}{c}}{1-\frac{m}{c}} \widehat{g}_{\frac{m}{c},t+1} - \widehat{R}_{t+1} \right] \Bigg\}. \end{aligned} \tag{21}$$

where:

$$\begin{aligned} \Omega &\equiv \frac{R\left(1-\left[(1-\gamma)\left(1-\frac{m}{c}\right)\right]\right) + \left[(1-\gamma)\left(1-\frac{m}{c}\right)\right]}{R\left(1-\left[(1-\gamma)\left(1-\frac{m}{c}\right)\right]\right)} \\ &= 1 + \frac{(1-\gamma)\left(1-\frac{m}{c}\right)}{R\left[1-(1-\gamma)\left(1-\frac{m}{c}\right)\right]} \geq 1. \end{aligned}$$

[3] The nominal interest rate and inflation both enter equation (20) with a one period lead. This is consistent with Carlstrom and Fuerst's (2001) "cash in advance timing" which contrasts with their "cash when I'm done timing", where both the nominal interest rate and inflation enter the Euler equation contemporaneously. Carlstrom and Fuerst (2001) reject the latter for CIA models.

The terms in $\hat{g}$ represent growth rates, therefore:

$$\widehat{g}_{t+1} = \ln\left(1+\overline{g}_{t+1}\right) - \ln\left(1+\overline{g}\right) \approx \overline{g}_{t+1} - \overline{g},$$

The Taylor condition (21) can now be expressed in net rates and absolute deviations from the BGP equilibrium, as demonstrated by the following proposition.

Proposition 1 *An equilibrium condition of the economy is in the form of the Taylor Rule (Orphanides, 2008) that sets deviations of the short-term nominal interest rate from some baseline path in proportion to deviations of target variables from their targets:*

$$\begin{aligned}\overline{R}_t - \overline{R} &= \Omega E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega\theta E_t\left(\overline{g}_{c,t+1} - \overline{g}\right) - \Omega\psi\left(1-\theta\right)E_t\overline{g}_{x,t+1} \\ &+ \frac{\left(1-\gamma\right)\left(1-\frac{m}{c}\right)}{R\left[1-\left(1-\gamma\right)\left(1-\frac{m}{c}\right)\right]}\left[\left(R-1\right)\frac{\frac{m}{c}}{1-\frac{m}{c}}E_t\overline{g}_{\frac{m}{c},t+1} - E_t\left(\overline{R}_{t+1}-\overline{R}\right)\right].\end{aligned} \tag{22}$$

where $\Omega \geq 1$, and for a given w, then $\frac{\partial\Omega}{\partial R} > 0$ and $\frac{\partial\Omega}{\partial A_F} > 0$, and the target values are equal to the balanced growth path equilibrium values.[4]

Proof. Since the BGP solution for normalized money demand is:

$$0 \leq \frac{m}{c} = 1 - A_F\left(\frac{\left(R-1\right)\gamma A_F}{w}\right)^{\frac{\gamma}{1-\gamma}} \leq 1,$$

then $\Omega \equiv 1 + \frac{(1-\gamma)\left(1-\frac{m}{c}\right)}{R\left[1-(1-\gamma)\left(1-\frac{m}{c}\right)\right]} \geq 1$ and, given w, $\frac{\partial\Omega}{\partial R} \geq 0$ and $\frac{\partial\Omega}{\partial A_F} \geq 0$. ■

For a linear production function of goods, w is the constant marginal product of labor, but more generally w is endogenous and will change; however this change in w is quantitatively small compared to changes in R and A_F, so that the derivatives above almost always hold true.

The term in $\bar{\pi}$ in equation (22) can be compared to the inflation target that features in many interest rate rules (e.g. Taylor, 1993; Clarida et al., 2000). This is usually set as an exogenous constant in an interest rate rule but represents the BGP rate of inflation in the Taylor condition.[5] The term in consumption growth is similar, but not identical, to the first difference of the output gap that features in the so-called 'speed limit' rule (Walsh, 2003). Alternatively, the term in the growth rate of leisure time can be compared to the unemployment rate which sometimes features in conventional interest rate rules in place of the output gap.[6]

Equation (22) also contains two terms which are not usually found in standard monetary policy reaction functions. Firstly, a term in the growth rate

[4] This is the the Brookings project form of the Taylor rule as described in Orphanides (2008).

[5] See Ireland (2007) for an example of a conventional interest rate rule with a time-varying inflation target.

[6] For example, Mankiw (2001) includes the unemployment rate in an interest rate rule and Rudebusch (2009) includes the 'unemployment gap'.

of (consumption normalized) money demand. Conventional interest rate rules are usually considered in the context of models which omit monetary relationships (e.g. Woodford, 2003) and thus money demand does not feature directly in the model, let alone the policy rule.[7] Secondly, the Taylor condition contains a term in the expected future nominal interest rate. This compares with the lagged nominal interest term which is often used to capture 'interest rate smoothing' in a conventional rule (e.g. Clarida et al., 2000).

In general the coefficient on inflation in (22) exceeds unity ($\Omega > 1$). This replicates the 'Taylor principle', whereby the nominal interest rate responds more than one-for-one to (expected future) inflation deviations from target. However, the inflation coefficient in the Taylor condition is a function of the BGP nominal interest rate (R), the consumption normalized demand for money (m/c) and the efficiency with which the banking sector transforms units of deposits into units of the credit service, as reflected by the magnitude of ($1-\gamma$). Furthermore, higher productivity in the banking sector (A_F) causes a higher velocity and implies a larger inflation coefficient in the Taylor condition. The magnitude of Ω clearly does not reflect a policymaker reaction to inflation in the conventional, 'reaction function', manner.[8]

Equation (22) can alternatively be rewritten in terms of the consumption velocity of money, $V_t \equiv \frac{c_t}{m_t}$, and the productive time ('employment') growth, $l \equiv 1 - x$. Using the fact that $\widehat{x}_t = -\frac{1-x}{x}\widehat{l}_t$:

$$\begin{aligned} \overline{R}_t - \overline{R} &= \Omega E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega\theta E_t\left(\overline{g}_{c,t+1} - \overline{g}\right) + \Omega\psi\left(1-\theta\right)\frac{l}{1-l}E_t\overline{g}_{l,t+1} \\ &\quad -\Omega_V E_t\overline{g}_{V,t+1} - \left(\Omega - 1\right)E_t\left(\overline{R}_{t+1} - \overline{R}\right). \end{aligned} \tag{23}$$

Where overbarred terms again denote net rates and where:

$$\Omega_V \equiv \frac{(R-1)}{R}\left(\frac{1-\gamma-(1-\gamma)\left(1-\frac{m}{c}\right)}{1-(1-\gamma)\left(1-\frac{m}{c}\right)}\right) = \frac{(R-1)}{R}\left(\frac{(1-\gamma)\frac{m}{c}}{\gamma+(1-\gamma)\frac{m}{c}}\right).$$

Proposition 2 *For the Taylor condition of equation (23), it is always true that* $0 \leq \Omega_V \leq 1 \leq \Omega$.

[7] Shifts in the demand for money are perfectly accommodated by adjustments to the money supply in order to maintain the rule-implied nominal interest rate. This, it is claimed, renders the evolution of the money supply an operational detail which need not be modelled directly (e.g. Woodford, 2008).

[8] Unlike Sørensen and Whitta-Jacobsen's (2005, pp.502-505) quantity theory based equilibrium condition, the inflation coefficient in (22) exceeds unity no matter what the interest elasticity of money demand. In their expression, the inflation coefficient falls below unity if the interest (semi) elasticity exceeds one in absolute value. In the Benk et al. (2010) model, the coefficient on inflation would exceed unity even in this case, although the central bank would not wish to increase the money supply growth rate to such an extent because seigniorage revenues would then begin to recede.

Proof.

$$\begin{aligned}
\Omega &\equiv \frac{R\left(1-(1-\gamma)\left(1-\frac{m}{c}\right)\right)+(1-\gamma)\left(1-\frac{m}{c}\right)}{R[1-(1-\gamma)\left(1-\frac{m}{c}\right)]} = 1+\frac{(1-\gamma)\left(1-\frac{m}{c}\right)}{R[1-(1-\gamma)\left(1-\frac{m}{c}\right)]}, \\
\Omega &= 1+\left(\frac{\omega}{R\left(1-\omega\right)}\right) \geq 1; \\
0 &\leq \omega \equiv (1-\gamma)\left(1-\frac{m}{c}\right) = (1-\gamma)\left[\gamma^{\gamma}\left(R-1\right)^{\gamma} w^{-\gamma} A_F\right]^{\frac{1}{1-\gamma}}, \\
\frac{m}{c} &= 1-A_F^{\frac{1}{1-\gamma}}\left[\frac{(R-1)\gamma}{w}\right]^{\frac{\gamma}{1-\gamma}} \leq 1, \\
1 &\geq 1-\gamma \geq \omega \geq 0. \\
\gamma+\omega &= \gamma+(1-\gamma)\left(1-\frac{m}{c}\right) = 1-\frac{m}{c}\left(1-\gamma\right) \leq 1. \\
&\Rightarrow \Omega_V \equiv \frac{(R-1)}{R}\left(\frac{1-\gamma-\omega}{1-\omega}\right) \leq 1. \\
&\Rightarrow 0 \leq \Omega_V \leq 1 \leq \Omega.
\end{aligned}$$

■

Note that at the Friedman (1969) optimum of $R = 1$, then $\frac{m}{c} = 1$, $\omega = 0$, and the velocity coefficient is $\Omega_V = 0$. The velocity growth term only matters when the nominal interest rate and inflation differ from the Friedman (1969) optimum and fluctuate. In turn, this has implications for $\Omega = 1 + \left(\frac{\omega}{R(1-\omega)}\right)$, since when $R = 1$, then $\omega = 0$, and $\Omega = 1$, as in the Fisher equation of interest rates. Only as $\frac{m}{c}$ falls below one (i.e. velocity rises above one), which is true for most practical experience, does the model's equivalent of the 'Taylor principle', $\Omega > 1$, hold.

Corollary 3 *Given w, then* $\frac{\partial\Omega}{\partial R} \geq 0$, $\frac{\partial\Omega_V}{\partial R} \geq 0$, $\frac{\partial\Omega}{\partial A_F} \geq 0$, $\frac{\partial\Omega_V}{\partial A_F} \leq 0$.

Proof. This comes directly from the definitions of parameters as given above. ■

A higher target R can be accomplished only by a higher BGP money supply growth rate. This would in turn make the inflation coefficient Ω higher, and so also the consumption growth coefficient $(\Omega\theta)$, and the forward interest rate and velocity coefficients would become more negative. A higher credit productivity factor A_F, and so a higher velocity, causes a higher inflation coefficient, and a more negative response to the forward-looking interest term but a less negative coefficient on the velocity growth term.

3.1 Misspecified Taylor Condition with Output Growth

It is not surprising to find that the growth rate of consumption appears in equation (23) rather than the output growth rate given that the derivation of the Taylor condition begins from the consumption Euler equation (20). However, the Taylor condition can be rewritten to include an output growth term and

thus better compare to standard Taylor rule specifications, especially the 'speed limit' rule considered by Walsh (2003). To derive this alternative rule, consider the fact that:

$$\begin{aligned} y_t &= c_t + i_t, \\ \widehat{y}_t &= \frac{c}{y}\widehat{c}_t + \frac{i}{y}\widehat{i}_t, \end{aligned}$$

and interpret $\widehat{i}_t$ using:

$$\widehat{i}_t = \frac{k}{i}\left[\widehat{k}_t - (1-\delta)\,\widehat{k}_{t-1}\right],$$

So that the growth rate of investment can be understood as the acceleration of the growth of capital gross of depreciation. The associated Taylor condition is:

$$\begin{aligned} \overline{R}_t - \overline{R} &= \Omega E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega\theta\left[\frac{y}{c}E_t\left(\overline{g}_{y,t+1} - \overline{g}\right) - \frac{i}{c}E_t\left(\overline{g}_{i,t+1} - \overline{g}\right)\right] \\ &\quad + \Omega\psi\left(1-\theta\right)\frac{l}{1-l}E_t\overline{g}_{l,t+1} - \Omega_V E_t\overline{g}_{V,t+1} - \left(\Omega - 1\right)E_t\left(\overline{R}_{t+1} - \overline{R}\right). \end{aligned} \tag{24}$$

A term in investment growth does not appear in standard, exogenously specified Taylor rules but plays a role as part of what is interpreted as growth in the output gap in this Taylor condition with output growth.

Equation (24) forms the basis for the two misspecified estimating equations considered in Section 5. The first misspecified estimating equation simply replaces the consumption growth term in equation (23) with an output gap term. As equation (24) shows, such an estimating equation erroneously overlooks the weighting on output ($\frac{y}{c}$) and omits the term in the investment growth rate. Replacing consumption growth with output growth without an additional term in investment therefore misrepresents the structure of the underlying, Benk et al. (2010), model.

The first misspecified estimating equation considered in Section 5 is therefore consistent with the following theoretical expression:

$$\begin{aligned} \overline{R}_t - \overline{R} &= \Omega E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega\theta\left[E_t\left(\overline{g}_{y,t+1} - \overline{g}\right)\right] \\ &\quad + \Omega\psi\left(1-\theta\right)\frac{l}{1-l}E_t\overline{g}_{l,t+1} - \Omega_V E_t\overline{g}_{V,t+1} - \left(\Omega - 1\right)E_t\left(\overline{R}_{t+1} - \overline{R}\right). \end{aligned} \tag{25}$$

Equation (25) does not accurately represent an equilibrium condition of the Benk et al. (2010) economy and, as such, is misspecified.

3.2 Misspecified Standard Taylor Rule

The second misspecified model erroneously imposes yet more restrictions on equation (24). Imposing the same restrictions used to reach equation (25) but also dropping the terms in productive time and velocity gives:

$$\begin{aligned} \overline{R}_t - \overline{R} &= \Omega E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega\theta\left[E_t\left(\overline{g}_{y,t+1} - \overline{g}\right)\right] \\ &\quad - \left(\Omega - 1\right)E_t\left(\overline{R}_{t+1} - \overline{R}\right). \end{aligned} \tag{26}$$

Preferences		
θ	1	Relative risk aversion parameter
ψ	1.84	Leisure weight
β	0.96	Discount factor
Goods Production		
α	0.64	Labor share in goods production
δ_K	0.031	Depreciation rate of goods sector
A_G	1	Goods productivity parameter
Human Capital Production		
ε	0.83	Labor share in human capital production
δ_H	0.025	Depreciation rate of human capital sector
A_H	0.21	Human capital productivity parameter
Banking Sector		
γ	0.11	Labor share in credit production
A_F	1.1	Banking productivity parameter
Government		
σ	0.05	Money growth rate

Table 1: Parameters

This can be interpreted as a conventional interest rate rule with an 'interest rate smoothing' term or, with the additional restriction that $\Omega = 1$, replicates an interest rate rule without smoothing. Once again, equation (26) does not accurately represent an equilibrium condition of the Benk et al. (2010) economy and is thus misspecified.

4 Calibration

We follow Benk et al. (2010) in using postwar U.S. data to calibrate the model (Table 1) and calculate a series of 'target values' (Table 2).

Subject to these values, we derive a set of theoretical magnitudes for the coefficients of the Taylor condition (23). These will subsequently be compared to the coefficients estimated from artificial data simulated from the model. Consider first the inflation coefficient (Ω): According to the calibration and target values presented in tables (1) and (2), its theoretical value is:

$$\begin{aligned}
\Omega &= 1 + \frac{\omega}{R(1-\omega)} = 1 + \frac{(1-\gamma)\left(1-\frac{m}{c}\right)}{R\left[1-(1-\gamma)\left(1-\frac{m}{c}\right)\right]} \\
&= 1 + \frac{(1-0.11)(1-0.38)}{1.0944\,(1-(1-0.11)(1-0.38))} \\
&= 2.125
\end{aligned}$$

g	0.024	Avg. annual output growth rate
π	0.026	Avg. annual inflation rate
R	0.0944	Nominal interest rate
l_G	0.248	Labor used in goods sector
l_H	0.20	Labor used in human capital sector
l_F	0.0018	Labor used in banking sector
i/y	0.238	Investment-output ratio in goods sector
m/c	0.38	Share of money transactions
x	0.55	Leisure time
$l \equiv 1 - x$	0.45	Productive time

Table 2: Target Values

And for $R = 1$, only cash is used so that $\frac{m}{c} = 1$, and Ω reverts to its lower bound of one. This similarly happens with zero credit productivity ($A_F = 0$), in which case only cash is used in exchange.

The remaining coefficients (except for velocity) are simple functions of the inflation coefficient. The consumption growth coefficient is $\Omega\theta$, which with $\theta = 1$ for log-utility should simply take the same magnitude as the coefficient on inflation ($\theta\Omega = 2.125$). With the leisure preference parameter calibrated at 1.84, and productive time ($1 - x \equiv l$) given as 0.45, the coefficient on the growth in productive time is:

$$\Omega\psi\left(1-\theta\right)\frac{l}{1-l} = \left(2.125\right)\left(1.84\right)\left(1-1\right)\frac{0.45}{0.55} = 0$$

Which equals zero because of the log utility specification but any underestimation in the model of θ can now be seen to be factored by $(2.125)\,(1.84)\,\frac{0.45}{0.55} = 3.199$. Given the magnitude of the inflation coefficient, the coefficient on the forward interest term is simply $-(\Omega - 1) = -1.125$ and the velocity coefficient is:

$$\begin{aligned}
-\Omega_V &\equiv -\frac{(R-1)}{R}\left(\frac{1-\gamma-\omega}{1-\omega}\right) \\
&= -\frac{(R-1)}{R}\left(\frac{1-\gamma-(1-\gamma)\left(1-\frac{m}{c}\right)}{\left[1-(1-\gamma)\left(1-\frac{m}{c}\right)\right]}\right) = \frac{(R-1)}{R}\left(\frac{(1-\gamma)\frac{m}{c}}{\left[1-(1-\gamma)\left(1-\frac{m}{c}\right)\right]}\right) \\
&= -\frac{(1.0944-1)}{1.0944}\left(\frac{(1-0.11)\,0.38}{(1-(1-0.11)\,(1-0.38))}\right) \\
&= -0.065
\end{aligned}$$

But would take a value of zero at the Friedman (1969) optimum ($R = 1$).

5 Artificial Data Estimation

The structural model which underpins the general equilibrium Taylor condition forms the basis for the data generating process of the simulated data. In partic-

ular, the Section 2 model is simulated using the Benk et al. (2010) calibration in order to generate 1000 alternative 'joint histories' for each of the variables in equation (23), where each history is 100 periods in length. To do so, 100 random sequences for the shock vector innovations are generated. Control functions of the log-linearized model are then used to compute sequences for each variable. Each observation within a given history may be thought of as an annual period given the frequency considered by the Benk et al. (2010) model; thus the data set comprises 1000, 100-year, samples of artificial data.

5.1 Estimation Methodology

Using conventional estimation techniques and the artificial dataset, this section presents the results of estimating the 'correctly specified' estimating equation based upon the 'true' theoretical relationship (23). In a similar manner, two misspecified estimating equations are evaluated using the same artificially generated data set.[9] Prior to implementing the various estimation procedures the simulated data is passed through three statistical filters: 1) a Hodrick-Prescott (HP) filter with a smoothing parameter selected according Ravn and Uhlig (2002); 2) a Christiano and Fitzgerald (2003) band pass filter which uses a 3-by-8 window with the intention of extracting the 'business cycle' component of the data; and 3) a Christiano and Fitzgerald (2003) band pass filter which uses a 2-by-15 window which retains more of the lower frequency trends, such as those highlighted by Comin and Gertler (2006). *A priori*, the 2-by-15 band pass filter may be the 'most relevant' to the underlying theoretical model; shocks in the model can cause low frequency events during the business cycle such as a change in the permanent income level without it returning to its previous level.It first filters the data using an HP filter and two alternative band pass filters.[10]

The first estimation technique considered is a simple OLS estimator, as used by Taylor (1999) in the context of a contemporaneous interest rate rule. However, because expected future terms feature in the estimating equation, we seek a suitable set of instruments for these terms.[11] Two instrumental variables (IV) techniques are considered and each differs by the instrument set employed. The first is a two stage least squares (2SLS) estimator under which the first lags of inflation, consumption growth, productive time growth and velocity growth and the second lag of the nominal interest rate (since the first lag is the dependent variable) are used as instruments. Adding a constant term to the instrument set forms an exactly identified 2SLS estimator (i.e. there are no over-identifying restrictions). In using lagged terms as instruments we exploit the fact that such terms are pre-determined and thus not susceptible to the aforementioned

[9] The exercise conducted here is similar to those conducted by Fève and Auray (2002), for a standard CIA model, and Salyer and Van Gaasbeck (2007), for a 'limited participation' model.

[10] Our filtering takes account of the Siklos and Wohar (2005) critique of estimation techniques for Taylor rules, which argues the need to account for stationary issues in the data.

[11] An innovation in the error term will cause the nominal interest rate to change which may, in turn, influence agents' expectations of the future state of the economy. For this reason, the error term and expected future variables are potentially correlated.

simultaneity problem. The 2SLS procedure applies a Newey-West adjustment for heteroskedasticity and autocorrelation (HAC) to the coefficient covariance matrix.

The second IV procedure is a generalized method of moments (GMM) estimator under which three additional lags of inflation, consumption growth, productive time growth and velocity growth and two further lags of the nominal interest rate are added to the instrument set.[12] Expanding the instrument set in this manner reduces the sample size available for each of the 1000 estimation runs but enables the validity of the instrument set to be assessed using the Hansen J-test. The GMM estimator employed applies a HAC adjustment to the estimation weighting matrix, using a Bartlett kernel with a Newey-West fixed bandwidth for this purpose, and iterates on the weighting matrix in two steps.[13] A similar HAC adjustment is also applied to the covariance weighting matrix.

The results are presented in three sets of tables, one set for each estimating equation, and are further divided according to the statistical filter applied to the data (giving nine tables in total). Alongside the estimates obtained from an 'unrestricted' estimating equation, each table also reports estimates obtained from a 'restricted' estimating equation which arbitrarily omits the forward interest rate term ($\beta_5 = 0$) in order to investigate the importance of this dynamic term. The tables present mean coefficient estimates along with the standard error of the coefficient estimates (as opposed to the mean standard error). The figures in square brackets report the number of coefficients estimated to be statistically different from zero at the 95% level of significance. We use this count as an indication of the 'precision' of the estimates. An 'adjusted mean' is also reported for each coefficient. This is calculated by setting non statistically significant coefficient estimates to zero. The tables also report mean R-square and mean adjusted R-square statistics, along with mean P-values for the F-statistic (these cannot be computed for the GMM estimator) and mean P-values for the Hansen J-statistic (if over-identifying restrictions are specified). The number of estimation runs for which the null hypothesis of the F-statistic is rejected is reported in round brackets and the number for which the null hypothesis of the J-statistic is *not* rejected is reported in braces.

5.2 General Taylor Condition

The first three tables present estimates obtained a 'correctly specified' estimating equation which accurately reflects the functional form of the Taylor condition (23) as follows:

[12] Carare and Tchaidze (2005, p.15) note that this 'four lags as instruments' specification is the standard approach in the interest rate rule literature (e.g. Orphanides, 2001). However, we opt to preserve the sample size of each estimation run rather than add an additional (fifth) lag of the expected future nominal interest rate to the GMM instrument set.

[13] Jondeau et al. (2004, p.227) state that: "To our knowledge, all estimations of the forward-looking reaction function based on GMM have so far relied on the two-step estimator." They proceed to consider more sophisticated GMM estimators but nevertheless identify advantages to the "simple approach" (p.238) adopted in the literature.

$$R_t = \beta_0 + \beta_1 E_t \pi_{t+1} + \beta_2 E_t g_{c,t+1} + \beta_3 E_t g_{l,t+1} + \beta_4 E_t g_{V,t+1} + \beta_5 E_t R_{t+1} + \varepsilon_t. \quad (27)$$

Expected future variables on the right hand side are obtained directly from the model simulation procedure described previously.[14]

Consider Table 5 for the 2-by-15 band pass filter.[15] Beginning with the unrestricted specification, the constant term is consistently estimated not to be significantly different from zero in a manner consistent with equation (23). The OLS and GMM procedures generate a greater number of statistically significant estimates than the 2SLS estimator for the remaining independent variables. For example, the 2SLS estimator provides a statistically significant estimate for the inflation coefficient for only 581 of the 1000 simulated histories while the OLS and GMM estimators both return 1000 significant estimates. A similar picture emerges for the other coefficient estimates in Table 5 and for the alternative filters (Tables 3 and 4).

Table 5 shows that the coefficient on inflation is estimated to be 2.176 using the OLS estimator and 2.304 using the GMM estimator.[16] These estimates compare favourably to the theoretical value of $\Omega = 2.125$. The coefficient on inflation is found to fall below unity when the the forward interest rate term is arbitrarily omitted from the estimating equation; a mean estimate of 0.614 is obtained from the OLS estimator and a mean estimate of 0.964 from the GMM estimator (the estimates are still produced to a high degree of 'precision'). Similar OLS and GMM estimates are obtained for the coefficient on inflation under the two alternative filters both in terms of the mean coefficient estimates for the unrestricted specification and in terms of the decline in magnitude when $\beta_5 = 0$ is imposed on the estimating equation.

Although the unrestricted specification yields estimates of the coefficient on inflation which are consistent with the theory, the implied estimates of the coefficient of relative risk aversion (θ) diverge somewhat from the calibration adopted ($\theta = 1$). Firstly, the estimated coefficient on inflation differs from the estimated coefficient on consumption growth, whereas equation (23) implies that these coefficients should take the same value when $\theta = 1$. Also, the coefficient on the growth rate of productive time is estimated to be non-zero for the vast majority of the 1000 estimation runs (996 and 1000 non-zero estimates under OLS and 2SLS respectively; Table 5) whereas its predicted value is zero.

The coefficient of relative risk aversion is not separately identified according to the estimation procedure but it can be estimated indirectly using the estimate of either β_2 or β_3. Firstly, using the GMM estimate the coefficient

[14] Empirical studies usually deal with expected future terms either by replacing them with realised future values and appealing to rational expectations for the resulting conditional forecast errors (e.g. Clarida et al., 1998, 2000) or by using private sector or central bank forecasts as empirical proxies for such terms (e.g. Orphanides, 2001; Siklos and Wohar, 2005).

[15] This is the filter that leaves more of the lower frequency information in the data.

[16] We focus on the OLS and GMM estimators because they produce more 'precise' estimates but also because the OLS estimator tends to reject the null hypothesis of the F-statistic more frequently than the 2SLS estimator (1000 vs. 904 rejections in Table 5, for example).

on consumption growth of 0.302 (Table 5) and the corresponding estimate for Ω, an implied estimate of θ can be calculated as $\frac{\beta_2}{\Omega} = \frac{0.302}{2.304} = 0.131$, which is smaller than the baseline calibration of $\theta = 1$. Alternatively, using the estimate of the coefficient on productive time growth, a second indirect estimate of θ can be obtained by replacing Ω with its estimate β_1 in the theoretical expression for β_3 as follows:

$$\begin{aligned} \beta_3 &= \beta_1 \psi (1-\theta) \frac{l}{1-l} \\ -0.359 &= (2.304)(1.84)(1-\theta) \frac{0.45}{0.55} \\ \theta &= 1.104 \end{aligned}$$

This gives an implied value of θ closer to the calibrated value.

Table 5 also reports that both the OLS and GMM procedures generate 1000 statistically significant estimates for the coefficient on velocity growth under the unrestricted estimating equation. The point estimates for this coefficient are -0.196 and -0.269 for OLS and GMM respectively; smaller than the theoretical prediction of -0.065. Similar estimates are obtained under the other two filters. Finally, Table 5 reports estimates of -1.758 (OLS) and -1.726 (GMM) for the forward interest rate coefficient compared to a theoretical value of -1.125. The estimates come reasonably close to the theoretical value but are slightly smaller than the prediction.

The key result is that Tables 3-5 consistently report an inflation coefficient which exceeds unity for the unrestricted estimating equation. This finding is found to be robust to the statistical filter applied to the data and to the estimator employed. This result also holds for the 'adjusted mean' under the 2SLS procedure which generally tends to yield fewer statistically significant estimates. The forward interest rate term is also found to be important in terms of generating a coefficient on inflation consistent with the underlying model.

The standard ('reaction function') interpretation of a coefficient on inflation in excess of unity is that policymakers adhere to a rule which reflects their intolerance towards inflation deviations from target. However, this interpretation is not applicable to the Taylor condition. The general result that the coefficient on inflation exceeds unity is a consequence of a money growth rule, not a measure of policymakers' attitude towards inflation.

5.3 Taylor Condition with Output Growth

The same estimation procedure is now applied to an estimating equation which replaces the term in the consumption growth rate in equation (27) with a term in output growth as follows:

$$R_t = \beta_0 + \beta_1 E_t \pi_{t+1} + \beta_2 E_t g_{y,t+1} + \beta_3 E_t g_{l,t+1} + \beta_4 E_t g_{V,t+1} + \beta_5 E_t R_{t+1} + \varepsilon_t. \quad (28)$$

This estimating equation corresponds to the misspecified representation of the Taylor condition with output growth; equation (25).

HP filtered data, where HP $\lambda = 6.25$	Unrestricted			Assumed $\beta_5 = 0$		
	OLS	2SLS	GMM	OLS	2SLS	GMM
β_0	-9.68E-07 [0]	-2.04E-07 [0]	8.09E-07 [17]	-7.60E-07 [0]	-4.78E-07 [0]	-7.57E-08 [8]
Standard error	2.87E-05	2.15E-05	3.15E-05	2.39E-05	1.67E-05	4.29E-05
Adjusted mean	-	-	-4.27E-08	-	-	2.59E-07
$E_t\pi_{t+1}$	2.019 [1000]	2.309 [691]	2.299 [1000]	0.315 [830]	0.757 [397]	0.621 [925]
Standard error	0.248	1.488	0.268	0.126	0.856	0.265
Adjusted mean	2.019	1.800	2.299	0.293	0.475	0.614
$E_t g_{c,t+1}$	0.251 [1000]	0.336 [959]	0.293 [1000]	0.172 [1000]	0.313 [989]	0.231 [1000]
Standard error	0.024	0.096	0.020	0.020	0.048	0.025
Adjusted mean	0.251	0.324	0.293	0.172	0.311	0.231
$E_t g_{l,t+1}$	-0.243 [890]	-0.536 [774]	-0.374 [997]	-0.281 [864]	-0.530 [774]	-0.427 [996]
Standard error	0.094	0.321	0.079	0.100	0.231	0.111
Adjusted mean	-0.236	-0.448	-0.374	-0.265	-0.453	-0.427
$E_t g_{V,t+1}$	-0.137 [990]	-0.267 [800]	-0.212 [1000]	-0.098 [889]	-0.317 [888]	-0.190 [992]
Standard error	0.031	0.228	0.033	0.036	0.109	0.056
Adjusted mean	-0.137	-0.229	-0.212	-0.093	-0.293	-0.190
$E_t R_{t+1}$	-1.819 [1000]	-2.338 [646]	-2.005 [1000]	N/A	N/A	N/A
Standard error	0.221	2.282	0.277			
Adjusted mean	-1.819	-1.692	-2.005			
$Mean$;						
R-square	0.789	<0	0.796	0.544	<0	0.482
Adjusted R-square	0.778	<0	0.785	0.525	<0	0.459
Pr(F-statistic)	2.35E-15 (1000)	0.015 (974)	N/A	3.93E-09 (1000)	0.003 (992)	N/A
Pr(J-statistic)	N/A	N/A	0.258 {1000}	N/A	0.159 {482}	0.269 {1000}
Sample size (1000x)	99	98	96	99	98	96

Notes:
· 'Standard error' measures the variation in the coefficient estimates.
· 'Adjusted mean' assigns a value of zero to non statistically significant estimates.
· F-statistic: null hypothesis of no joint significance of the independent variables (not available under GMM).
· J-statistic: null hypothesis that the instrument set is valid (only available if there are over-identifying restrictions).
· [ ] records the number of statistically significant coefficient estimates, () the number of F-statistic rejections, and {} the number of J-statistic non-rejections (all at the 5% level of significance).

Table 3: Taylor Condition Estimation, HP Filtered Data, Ravn and Uhlig (2002) Smoothing Parameter, 100 Years Simulated, 1000 Estimations Average.

BP Filter,	Unrestricted			Assumed $\beta_5 = 0$		
3-by-8 Window	OLS	2SLS	GMM	OLS	2SLS	GMM
β_0	-8.09E-07 [0]	-3.35E-05 [0]	-9.38E-07 [1]	-7.32E-07 [0]	-3.06E-06 [0]	-1.28E-06 [1]
Standard error	1.77E-05	0.001	2.05E-05	1.48E-05	4.41E-05	2.65E-05
Adjusted mean	-	-	-5.51E-08	-	-	-6.11E-08
$E_t\pi_{t+1}$	2.175 [998]	4.414 [729]	2.424 [1000]	0.634 [969]	2.416 [964]	0.684 [977]
Standard error	0.383	18.203	0.300	0.196	1.110	0.221
Adjusted mean	2.174	2.151	2.424	0.629	2.283	0.681
$E_t g_{c,t+1}$	0.283 [1000]	0.476 [628]	0.314 [1000]	0.155 [1000]	0.174 [832]	0.168 [1000]
Standard error	0.043	3.879	0.027	0.029	0.074	0.030
Adjusted mean	0.283	0.234	0.314	0.155	0.159	0.168
$E_t g_{l,t+1}$	-0.238 [833]	-0.515 [440]	-0.312 [983]	-0.222 [687]	-0.597 [676]	-0.268 [867]
Standard error	0.130	13.195	0.099	0.133	0.360	0.134
Adjusted mean	-0.230	-0.196	-0.311	-0.195	-0.458	-0.258
$E_t g_{V,t+1}$	-0.152 [982]	-0.282 [354]	-0.174 [999]	-0.174 [974]	-0.603 [972]	-0.194 [984]
Standard error	0.043	9.188	0.039	0.052	0.249	0.057
Adjusted mean	-0.152	-0.127	-0.174	-0.173	-0.577	-0.194
$E_t R_{t+1}$	-2.034 [994]	-4.584 [422]	-2.289 [1000]	N/A	N/A	N/A
Standard error	0.425	59.596	0.341			
Adjusted mean	-2.032	-1.215	-2.289			
$Mean$;						
R-square	0.791	<0	0.842	0.576	<0	0.590
Adjusted R-square	0.779	<0	0.833	0.558	<0	0.572
Pr(F-statistic)	9.08E-10 (1000)	0.078 (877)	N/A	2.31E-07 (1000)	0.005 (980)	N/A
Pr(J-statistic)	N/A	N/A	0.214 {1000}	N/A	0.340 {847}	0.248 {1000}
Sample size (1000x)	99	98	96	99	98	96

Notes:
· 'Standard error' measures the variation in the coefficient estimates.
· 'Adjusted mean' assigns a value of zero to non statistically significant estimates.
· F-statistic: null hypothesis of no joint significance of the independent variables (not available under GMM).
· J-statistic: null hypothesis that the instrument set is valid (only available if there are over-identifying restrictions).
· [ ] reports the number of statistically significant coefficient estimates, () reports the number of F-statistic rejections, and {} reports the number of J-statistic non-rejections (all at the 5% level of significance).

Table 4: Taylor Condition Estimation, Band Pass Filtered Data (3-by-8 years), 100 Years Simulated, 1000 Estimations Average.

BP Filter,	Unrestricted			Assumed $\beta_5 = 0$		
2-by-15 Window	OLS	2SLS	GMM	OLS	2SLS	GMM
β_0	-2.48E-06 [0]	-2.95E-06 [0]	6.72E-07 [19]	-1.78E-06 [0]	-3.07E-06 [0]	3.09E-06 [17]
Standard error	4.35E-05	1.40E-04	4.81E-05	3.35E-05	4.84E-05	6.31E-05
Adjusted mean	-	-	1.34E-07	-	-	3.19E-07
$E_t\pi_{t+1}$	2.176 [1000]	1.970 [581]	2.304 [1000]	0.614 [999]	1.132 [770]	0.964 [999]
Standard error	0.195	3.736	0.272	0.107	0.641	0.168
Adjusted mean	2.176	1.409	2.304	0.613	0.942	0.964
$E_t g_{c,t+1}$	0.277 [1000]	0.366 [729]	0.302 [1000]	0.170 [1000]	0.261 [846]	0.207 [1000]
Standard error	0.016	0.842	0.025	0.017	0.103	0.026
Adjusted mean	0.277	0.264	0.302	0.170	0.228	0.207
$E_t g_{l,t+1}$	-0.294 [996]	-0.428 [533]	-0.359 [1000]	-0.210 [737]	-0.262 [405]	-0.277 [932]
Standard error	0.068	1.231	0.084	0.088	0.205	0.111
Adjusted mean	-0.294	-0.245	-0.359	-0.182	-0.151	-0.271
$E_t g_{V,t+1}$	-0.196 [1000]	-0.360 [809]	-0.269 [1000]	-0.158 [998]	-0.307 [941]	-0.237 [1000]
Standard error	0.024	0.356	0.031	0.032	0.078	0.042
Adjusted mean	-0.196	-0.274	-0.269	-0.158	-0.291	-0.237
$E_t R_{t+1}$	-1.758 [1000]	-1.415 [338]	-1.726 [1000]	N/A	N/A	N/A
Standard error	0.199	8.635	0.320			
Adjusted mean	-1.758	-0.722	-1.726			
$Mean$;						
R-square	0.829	<0	0.782	0.625	<0	0.521
Adjusted R-square	0.820	<0	0.770	0.609	<0	0.500
Pr(F-statistic)	2.64E-23 (1000)	0.051 (904)	N/A	5.12E-10 (1000)	0.004 (985)	N/A
Pr(J-statistic)	N/A	N/A	0.319 {1000}	N/A	0.300 {753}	0.302 {1000}
Sample size (1000x)	99	98	96	99	98	96

Notes:

· 'Standard error' measures the variation in the coefficient estimates.

· 'Adjusted mean' assigns a value of zero to non statistically significant estimates.

· F-statistic: null hypothesis of no joint significance of the independent variables (not available under GMM).

· J-statistic: null hypothesis that the instrument set is valid (only available if there are over-identifying restrictions).

· [ ] reports the number of statistically significant coefficient estimates, () reports the number of F-statistic rejections, and {} reports the number of J-statistic non-rejections (all at the 5% level of significance).

Table 5: Taylor Condition Estimation, Band Pass Filtered Data (2-by-15 years), 100 Years Simulated, 1000 Estimations Average.

The instrument sets used for the 2SLS and GMM estimators are modified by replacing consumption growth with output growth but remain unchanged in terms of the number of lags included. This alternative specification may be thought of as the Taylor condition equivalent of a 'speed limit' interest rate rule (e.g. Walsh, 2003). However, the simulated data remains unchanged therefore equation (28) represents a misspecified version of the 'correct' estimating, which remains equation (27), as before.

Tables 6-8 present the estimation results obtained from equation (28). Comparing these results to those presented in Tables 3-5, there is a general decline in the precision with which the coefficients are estimated and in the number of rejections of the null hypothesis of the F-statistic for joint significance. This is unsurprising given that we have introduced an element of misspecification into the estimating equation (although the GMM estimates for HP filtered data, table 6, are still estimated very precisely).

The estimated coefficients on inflation are now substantially greater than the coefficients obtained from the 'correctly specified' estimating equation (27). For instance, the GMM estimate for the unrestricted estimating equation rises from 2.304 in Table 5 to 5.307 in Table 8 (or 5.277 according to the adjusted mean). Similarly, the OLS estimate increases from 2.176 to 4.128 (or 4.088 adjusted). Corresponding upward shifts in the estimated inflation coefficient can be found when comparing Table 4 to Table 7 for the 3x8 bandpass filter and even larger increases are found for the HP filtered data (from Table 3 to Table 6). Therefore, the estimates diverge further from the theoretical value of $\Omega = 2.125$ under this misspecified estimating equation.

The alteration to the estimating equation also induces a substantial decrease in the estimated coefficients on the terms in the productive time growth rate and the forward nominal interest rate. The estimates for these coefficients tend to diverge further away from the theoretical predictions of the model. The estimated coefficient on the productive time growth rate decreases from -0.294 to -2.052 between Table 5 and Table 8 according to the OLS estimator and from -0.359 to -2.838 for the GMM estimator; the theoretical prediction is that $\beta_3 = 0$ ($\theta = 1$). The GMM estimates of the forward interest rate term also decrease from -2.005, -2.289 and -1.726 under the HP filter, BP(3x8) and BP(2x15) filters respectively to -12.868, -6.308 and -4.532. Once again, the estimates diverge further from theoretical value of -1.125. Analogous results are obtained for the OLS estimator.

The estimated coefficients for output growth in Tables 6-8 are similar to those for consumption growth presented in Tables 3-5, despite the impact the misspecified estimating equation has on the other estimates. The OLS estimate for β_3, for example, is 0.296 (0.292 adjusted) in Table 8 compared to the corresponding estimate of 0.277 found in Table 5. For the GMM estimator the coefficient on output growth is 0.410 (0.406 adjusted) in Table 8 compared to the corresponding estimate of 0.302 reported in Table 5.

The velocity growth term is estimated precisely by the GMM estimator even after the modification to the estimating equation. Estimates of β_4 retain the correct sign and are of a similar magnitude as under the correctly specified

estimating equation (for example, a GMM estimate of -0.190 in Table 8 as compared to a corresponding estimate of -0.269 in Table 5).

Considering the restricted specification ($\beta_5 = 0$), the estimates undergo similar changes as those for the restricted version of the 'correct' estimating equation (27). The OLS and GMM estimators generate inflation coefficients which fall below unity in a manner incompatible with the theoretical model from which the Taylor condition is derived (although there is one exception to this for the GMM estimator in Table 8).

The estimates obtained from applying equation (28) to the simulated data show that adapting the estimating equation in a seemingly minor way can have a substantial impact upon the reported estimates. The erratic results obtained from this misspecified estimating equation provide an illustration of the fundamental difference between the Taylor condition and a conventional interest rate rule. Unlike a Taylor rule, the Taylor condition cannot be modified in an *ad hoc* fashion.[17] In order to make the progression from (27) to (28) in a legitimate manner, one would need to alter the underlying model in some way. Section 6 presents a restricted version of the general Benk et al. (2010) model for which equation (28) would be the appropriate estimating equation. New simulated data would then need to be generated from this alternative model prior to re-estimation.

5.4 A Conventional Interest Rate Rule

The estimation procedure is now re-applied to the following estimating equation:

$$R_t = \beta_0 + \beta_1 E_t \pi_{t+1} + \beta_2 E_t g_{y,t+1} + \beta_5 E_t R_{t+1} + \varepsilon_t. \tag{29}$$

This estimating equation corresponds to the misspecified representation of the Taylor condition with output growth; equation (26).[18] Notably, the term in velocity growth is absent from this expression. This omission would be expected to have a bearing on the estimates because equations (27) and (28) produced a high number of statistically significant estimates for this velocity term.

Tables 9-11 present the estimates generated from applying (29) to the simulated data. The estimates are generally found to be poor in terms of the number of statistically significant cases produced. This is unsurprising given that we have added yet another source of misspecification to the estimating equation. The estimated coefficient on inflation does not exceed unity for any of the three filters considered according to the adjusted mean (which is the appropriate figure to consult given the low number of statistically significant estimates). The results are also comparatively weak in terms of the frequency

[17] Clarida et al. (1998), for example, add the exchange rate to the standard Taylor rule and Cecchetti et al. (2000) and Bernanke and Gertler (2001) consider whether policymakers should react to asset prices.

[18] The instrument set now comprises of four lags of expected future inflation, four lags of expected future output growth, the second, third and fourth lags of the nominal interest rate and a constant term for the GMM estimator or just the shortest of these lags and a constant term for the exactly identified 2SLS estimator.

HP filtered data, where HP $\lambda = 6.25$	Unrestricted			Assumed $\beta_5 = 0$		
	OLS	2SLS	GMM	OLS	2SLS	GMM
β_0	-1.20E-06 [0]	-2.43E-06 [0]	5.08E-06 [39]	-2.31E-07 [0]	-2.05E-07 [0]	2.18E-06 [1]
Standard error	3.75E-05	1.60E-04	9.03E-05	1.19E-05	4.94E-05	5.72E-05
Adjusted mean	-	-	1.03E-06	-	-	1.64E-07
$E_t\pi_{t+1}$	4.838 [903]	15.555 [655]	12.570 [1000]	0.040 [156]	1.057 [268]	0.151 [579]
Standard error	3.086	53.295	2.368	0.185	3.929	0.530
Adjusted mean	4.762	11.697	12.570	0.021	0.263	0.141
$E_t g_{y,t+1}$	0.343 [941]	1.501 [738]	0.954 [1000]	0.023 [225]	0.260 [359]	0.049 [644]
Standard error	0.221	5.000	0.176	0.022	0.523	0.052
Adjusted mean	0.340	1.093	0.954	0.011	0.119	0.046
$E_t g_{l,t+1}$	-2.360 [877]	-10.677 [716]	-6.855 [1000]	-0.125 [128]	-1.861 [323]	-0.333 [549]
Standard error	1.618	36.513	1.297	0.235	3.798	0.468
Adjusted mean	-2.317	-7.642	-6.855	-0.050	-0.754	-0.297
$E_t g_{V,t+1}$	-0.037 [328]	-0.320 [355]	-0.180 [940]	0.005 [108]	-0.157 [214]	-0.007 [505]
Standard error	0.054	2.003	0.074	0.043	0.512	0.093
Adjusted mean	-0.029	-0.149	-0.178	0.002	-0.035	-0.006
$E_t R_{t+1}$	-4.975 [910]	-16.578 [639]	-12.868 [1000]	N/A	N/A	N/A
Standard error	3.151	54.286	2.381			
Adjusted mean	-4.907	-11.911	-12.868			
$Mean$						
R-square	0.270	<0	0.201	0.063	<0	<0
Adjusted R-square	0.231	<0	0.156	0.023	<0	<0
Pr(F-statistic)	0.033 (855)	0.192 (579)	N/A	0.325 (203)	0.283 (261)	N/A
Pr(J-statistic)	N/A	N/A	0.305 {1000}	N/A	0.111 {286}	0.222 {1000}
Sample size (1000x)	99	98	96	99	98	96

Notes:
· 'Standard error' measures the variation in the coefficient estimates.
· 'Adjusted mean' assigns a value of zero to non statistically significant estimates.
· F-statistic: null hypothesis of no joint significance of the independent variables (not available under GMM).
· J-statistic: null hypothesis that the instrument set is valid (only available if there are over-identifying restrictions).
· [ ] reports the number of statistically significant coefficient estimates, () reports the number of F-statistic rejections and {} reports the number of J-statistic non-rejections (all at the 5% level of significance).

Table 6: Output Growth instead of Consumption Growth, HP Filtered Data, Ravn and Uhlig (2002) Smoothing Parameter, 100 Years Simulated, 1000 Estimations Average.

BP Filter,	Unrestricted			Assumed $\beta_5 = 0$		
3-by-8 Window	OLS	2SLS	GMM	OLS	2SLS	GMM
β_0	-1.34E-06 [0]	-9.27E-05 [0]	-8.05E-07 [0]	-6.38E-07 [0]	-7.97E-06 [0]	-8.52E-07 [1]
Standard error	1.88E-05	0.003	3.45E-05	1.28E-05	8.21E-05	2.90E-05
Adjusted mean	-	-	-	-	-	-1.22E-07
$E_t\pi_{t+1}$	3.471 [710]	5.556 [186]	6.627 [956]	0.735 [926]	3.859 [935]	0.815 [953]
Standard error	3.298	340.051	3.920	0.258	1.990	0.308
Adjusted mean	3.289	3.677	6.591	0.719	3.469	0.810
$E_t g_{y,t+1}$	0.233 [633]	0.275 [143]	0.464 [926]	0.036 [517]	0.151 [568]	0.043 [719]
Standard error	0.243	24.979	0.287	0.026	0.107	0.033
Adjusted mean	0.219	0.221	0.460	0.028	0.101	0.041
$E_t g_{l,t+1}$	-1.651 [633]	-2.572 [161]	-3.277 [934]	-0.306 [460]	-1.722 [715]	-0.389 [685]
Standard error	1.706	171.157	2.012	0.259	1.150	0.311
Adjusted mean	-1.555	-1.721	-3.249	-0.230	-1.296	-0.355
$E_t g_{V,t+1}$	-0.146 [840]	-0.659 [362]	-0.169 [916]	-0.146 [833]	-0.842 [929]	-0.163 [897]
Standard error	0.068	13.117	0.078	0.066	0.465	0.079
Adjusted mean	-0.140	-0.256	-0.166	-0.139	-0.745	-0.160
$E_t R_{t+1}$	-2.974 [546]	-3.250 [121]	-6.308 [874]	N/A	N/A	N/A
Standard error	3.533	367.537	4.209			
Adjusted mean	-2.795	-2.574	-6.199			
$Mean$;						
R-square	0.341	<0	0.386	0.263	<0	0.254
Adjusted R-square	0.305	<0	0.352	0.232	<0	0.221
Pr(F-statistic)	0.006 (970)	0.300 (518)	N/A	0.014 (937)	0.050 (834)	N/A
Pr(J-statistic)	N/A	N/A	0.211 {1000}	N/A	0.337 {832}	0.248 {1000}
Sample size (1000x)	99	98	96	99	98	96

Notes:
· 'Standard error' measures the variation in the coefficient estimates.
· 'Adjusted mean' assigns a value of zero to non statistically significant estimates.
· F-statistic: null hypothesis of no joint significance of the independent variables (not available under GMM).
· J-statistic: null hypothesis that the instrument set is valid (only available if there are over-identifying restrictions).
· [ ] reports the number of statistically significant coefficient estimates, () reports the number of F-statistic rejections and {} reports the number of J-statistic non-rejections (all at the 5% level of significance).

Table 7: Output Growth instead of Consumption Growth, Band Pass Filtered data (3-by-8 years), 100 Years Simulated, 1000 Estimations Average.

BP Filter,	Unrestricted			Assumed $\beta_5 = 0$		
2-by-15 Window	OLS	2SLS	GMM	OLS	2SLS	GMM
β_0	-3.69E-06 [0]	1.05E-04 [0]	3.22E-06 [13]	-8.79E-07 [0]	-4.94E-06 [0]	2.71E-06 [5]
Standard error	6.46E-05	0.004	8.79E-05	2.49E-05	9.61E-05	6.60E-05
Adjusted mean	-	-	-4.28E-08	-	-	9.02E-08
$E_t\pi_{t+1}$	4.128 [968]	12.191 [187]	5.307 [963]	0.542 [941]	2.346 [882]	1.104 [990]
Standard error	1.676	228.990	2.568	0.170	1.176	0.263
Adjusted mean	4.088	2.474	5.277	0.532	2.014	1.103
$E_t g_{y,t+1}$	0.296 [960]	1.072 [203]	0.410 [940]	0.038 [565]	0.210 [268]	0.082 [957]
Standard error	0.122	21.681	0.204	0.020	0.317	0.029
Adjusted mean	0.292	0.238	0.406	0.029	0.079	0.081
$E_t g_{l,t+1}$	-2.052 [950]	-7.469 [205]	-2.838 [951]	-0.283 [423]	-1.459 [356]	-0.613 [943]
Standard error	0.877	139.821	1.415	0.189	1.782	0.233
Adjusted mean	-2.020	-1.607	-2.814	-0.190	-0.665	-0.602
$E_t g_{V,t+1}$	-0.117 [882]	-0.382 [318]	-0.191 [967]	-0.095 [733]	-0.247 [590]	-0.158 [929]
Standard error	0.042	5.355	0.064	0.043	0.164	0.061
Adjusted mean	-0.112	-0.119	-0.190	-0.084	-0.162	-0.156
$E_t R_{t+1}$	-3.784 [893]	-11.475 [122]	-4.532 [860]	N/A	N/A	N/A
Standard error	1.772	255.026	2.784			
Adjusted mean	-3.656	-1.714	-4.421			
$Mean$;						
R-square	0.358	<0	0.122	0.246	<0	<0
Adjusted R-square	0.324	<0	0.073	0.214	<0	<0
Pr(F-statistic)	0.001 (995)	0.374 (420)	N/A	0.020 (920)	0.055 (826)	N/A
Pr(J-statistic)	N/A	N/A	0.227 {1000}	N/A	0.261 {685}	0.265 {1000}
Sample size (1000x)	99	98	96	99	98	96

Notes:
· 'Standard error' measures the variation in the coefficient estimates.
· 'Adjusted mean' assigns a value of zero to non statistically significant estimates.
· F-statistic: null hypothesis of no joint significance of the independent variables (not available under GMM).
· J-statistic: null hypothesis that the instrument set is valid (only available if there are over-identifying restrictions).
· [ ] reports the number of statistically significant coefficient estimates, () reports the number of F-statistic rejections and {} reports the number of J-statistic non-rejections (all at the 5% level of significance).

Table 8: Output Growth instead of Consumption Growth, Band Pass Filtered data (2-by-15 years), 100 Years Simulated, 1000 Estimations Average.

with which the null hypothesis of the F-statistic is rejected (the HP filtered data gives particularly weak results in this regard) and in terms of the number of non-rejections of the null hypothesis of the Hansen J-test. The latter finding calls into question the validity of the standard instrument set used for the GMM estimator under equation (29).

The estimation results are similarly imprecise under the restricted specification ($\beta_5 = 0$) for the HP filter, although we now obtain a reasonable number of significant estimates for the coefficient on inflation for the OLS and GMM estimators in Tables 10 and 11. These estimates are similar for the 3-by-8 BP filter, 0.227 (adjusted mean, OLS) and 0.283 (adjusted mean, GMM), but differ quite substantially for the 2-by-15 filter (0.317 compared to 0.894).

In short, imposing a conventional Taylor rule restricts the 'true' estimating equation to such an extent that the theoretical prediction that the coefficient on expected inflation exceeds unity is not recovered even under the unrestricted estimating equation. Results such as these derived from an estimating equation such as (29) might erroneously be interpreted to signify that the Taylor principle is violated but this result is simply a product of a misspecified estimating equation in the present context. Section 6 presents a restricted version of the general Benk et al. (2010) model for which equation (29) would be the appropriate estimating equation. Again, new simulated data would then need to be generated from this alternative model prior to re-estimation.

6 Alternative Taylor Conditions

We now consider alternative ways in which to express the Taylor condition derived from the full model (23) and how the functional form of the Taylor condition changes when the underlying (Benk et al., 2010) model is modified, i.e. restricted, in various ways.

First, we derive a 'backward looking' Taylor condition from equation (23) and secondly a Taylor condition which incorporates the growth rate of exchange credit. Third, we progress to consider restricted versions of the general Benk et al. (2010) model and demonstrate how the Taylor principle is lost as a theoretical result in a cash only economy. Fourth, we consider an economy without physical capital which implies that the output growth term replaces the consumption growth term ($y = c$). If we further deny consumers use of exchange credit and eliminate leisure preference then a standard interest rate rule emerges as an equilibrium condition of the model but with the theoretical prediction that the coefficient on inflation is one. Finally, we consider an exogenous growth version of the model in which human capital investment is eliminated from the model. In contrast to equation (23), the 'inflation target' and 'potential output' terms which feature in the resulting Taylor condition are now exogenous.

For each of these restricted versions of the model we could run the same experiment of simulating the model and evaluating artificial data using the various estimation techniques applied in Section 5. It would still be the case that the coefficient estimates that emerge from this process are not the result of

HP filtered data, where HP $\lambda = 6.25$	Unrestricted			Assumed $\beta_5 = 0$		
	OLS	2SLS	GMM	OLS	2SLS	GMM
β_0	-5.03E-07 [0]	-4.06E-04 [0]	2.54E-06 [1]	-3.26E-07 [0]	-8.02E-07 [0]	7.55E-07 [2]
Standard error	1.02E-05	0.009	7.01E-05	9.91E-06	2.91E-05	6.42E-05
Adjusted mean	-	-	-1.68E-07	-	-	2.65E-08
$E_t\pi_{t+1}$	0.497 [268]	54.929 [43]	0.503 [422]	0.052 [68]	-0.499 [172]	0.113 [551]
Standard error	0.485	1882.477	1.230	0.089	1.723	0.336
Adjusted mean	0.288	-0.157	0.447	0.012	0.045	0.106
$E_t g_{y,t+1}$	0.024 [365]	-1.547 [122]	0.026 [453]	0.012 [198]	0.151 [721]	0.021 [484]
Standard error	0.017	40.707	0.022	0.011	0.105	0.021
Adjusted mean	0.015	0.027	0.020	0.005	0.112	0.017
$E_t R_{t+1}$	-0.489 [233]	-54.930 [35]	-0.462 [420]	N/A	N/A	N/A
Standard error	0.506	1764.434	1.303			
Adjusted mean	-0.263	0.153	-0.402			
$Mean$;						
R-square	0.059	<0	<0	0.033	<0	<0
Adjusted R-square	0.029	<0	<0	0.013	<0	<0
Pr(F-statistic)	0.274 (281)	0.626 (61)	N/A	0.323 (163)	0.084 (653)	N/A
Pr(J-statistic)	N/A	N/A	0.029 {94}	N/A	0.087 {235}	0.040 {212}
Sample size (1000x)	99	98	96	99	98	96

Notes:
· 'Standard error' measures the variation in the coefficient estimates.
· 'Adjusted mean' assigns a value of zero to non statistically significant estimates.
· F-statistic: null hypothesis of no joint significance of the independent variables (not available under GMM).
· J-statistic: null hypothesis that the instrument set is valid (only available if there are over-identifying restrictions).
· [ ] reports the number of statistically significant coefficient estimates, () reports the number of F-statistic rejections and {} reports the number of J-statistic non-rejections (all at the 5% level of significance).

Table 9: Output Growth in a Standard Taylor Rule, HP Filtered Data, Ravn and Uhlig (2002) Smoothing Parameter, 100 Years Simulated, 1000 Estimations Average.

BP Filter,	Unrestricted			Assumed $\beta_5 = 0$		
3-by-8 Window	OLS	2SLS	GMM	OLS	2SLS	GMM
β_0	-4.75E-07 [0]	-2.12E-05 [0]	-5.80E-07 [0]	-4.06E-07 [0]	-1.96E-06 [0]	-9.56E-07 [0]
Standard error	1.33E-05	3.47E-04	3.95E-05	1.33E-05	5.33E-05	4.16E-05
Adjusted mean	-	-	-	-	-	-
$E_t\pi_{t+1}$	0.315 [284]	2.267 [282]	0.352 [398]	0.254 [800]	-0.051 [113]	0.285 [957]
Standard error	0.594	9.395	0.739	0.083	0.817	0.099
Adjusted mean	0.223	0.557	0.291	0.227	0.062	0.283
$E_t g_{y,t+1}$	0.018 [335]	0.524 [544]	0.020 [401]	0.017 [394]	0.285 [743]	0.019 [495]
Standard error	0.025	2.350	0.030	0.018	0.344	0.022
Adjusted mean	0.013	0.188	0.015	0.013	0.188	0.016
$E_t R_{t+1}$	-0.074 [217]	-2.915 [243]	-0.080 [344]	N/A	N/A	N/A
Standard error	0.620	11.221	0.768			
Adjusted mean	-0.056	-0.555	-0.076			
$Mean$;						
R-square	0.135	<0	0.115	0.112	<0	0.100
Adjusted R-square	0.108	<0	0.086	0.093	<0	0.081
Pr(F-statistic)	0.047 (792)	0.268 (441)	N/A	0.039 (805)	0.074 (732)	N/A
Pr(J-statistic)	N/A	N/A	0.042 {256}	N/A	0.226 {613}	0.056 {461}
Sample size (1000x)	99	98	96	99	98	96

Notes:
· 'Standard error' measures the variation in the coefficient estimates.
· 'Adjusted mean' assigns a value of zero to non statistically significant estimates.
· F-statistic: null hypothesis of no joint significance of the independent variables (not available under GMM).
· J-statistic: null hypothesis that the instrument set is valid (only available if over-identifying restrictions).
· [ ] records the number of statistically significant coefficient estimates, () the number of F-statistic rejections and {} the number of J-statistic non-rejections, at 5% level of significance.

Table 10: Output Growth in a Standard Taylor Rule, Band Pass Filtered Data (3-by-8 years), 100 Years Simulated, 1000 Estimations Average.

BP Filter,	Unrestricted			Assumed $\beta_5 = 0$		
2-by-15 Window	OLS	2SLS	GMM	OLS	2SLS	GMM
β_0	-6.03E-07 [0]	-8.11E-05 [0]	-2.84E-06 [7]	-5.32E-07 [0]	1.85E-07 [0]	-2.73E-07 [11]
Standard error	2.60E-05	0.008	1.16E-04	2.59E-05	2.98E-04	1.13E-04
Adjusted mean	-	-	-1.12E-06	-	-	4.92E-07
$E_t\pi_{t+1}$	0.311 [238]	12.975 [21]	0.142 [349]	0.327 [929]	1.493 [412]	0.895 [980]
Standard error	0.446	480.114	0.950	0.099	4.468	0.315
Adjusted mean	0.185	0.020	0.110	0.317	0.814	0.894
$E_t g_{y,t+1}$	0.020 [281]	-0.666 [40]	0.019 [236]	0.021 [404]	0.027 [162]	0.031 [456]
Standard error	0.017	33.878	0.027	0.012	0.825	0.027
Adjusted mean	0.011	0.012	0.012	0.013	0.041	0.024
$E_t R_{t+1}$	0.007 [140]	-5.714 [27]	0.909 [524]	N/A	N/A	N/A
Standard error	0.473	387.875	0.999			
Adjusted mean	0.010	0.101	0.781			
$Mean$;						
R-square	0.168	<0	<0	0.153	<0	<0
Adjust R-square	0.142	<0	<0	0.136	<0	<0
Pr(F-statistic)	0.030 (882)	0.527 (162)	N/A	0.025 (889)	0.146 (602)	N/A
Pr(J-statistic)	N/A	N/A	0.050 {340}	N/A	0.351 (677)	0.058 (448)
Sample size (1000x)	99	98	96	99	98	96

Notes:
· 'Standard error' measures the variation in the coefficient estimates.
· 'Adjusted mean' assigns a value of zero to non statistically significant estimates.
· F-statistic: null hypothesis of no joint significance of the independent variables (not available under GMM).
· J-statistic: null hypothesis that the instrument set is valid (only available if over-identifying restrictions).
· [ ] records the number of statistically significant coefficient estimates, () the number of F-statistic rejections and {} the number of J-statistic non-rejections, at 5% level of significance.

Table 11: Output Growth in a Standard Taylor Rule, Band Pass Filtered Data (2-by-15 years), 100 Years Simulated, 1000 Estimations Average.

either an 'aggressive' or a 'passive' reaction to inflation by policymakers. The general principle is to derive the precise form of the Taylor condition from a particular structural, general equilibrium model so that we can obtain an *a priori* understanding of the potential implications of a misspecified estimating equation for the coefficients obtained when taking such expressions to actual data.

6.1 Backward Looking Taylor Condition

Mathematically, the Taylor condition can also be formulated to have a lagged dependent on the right hand side instead of the lead independent variable which appears in equation (23). This yields a similar expression written in terms of R_{t+1} instead of R_t:

$$\begin{aligned} \overline{R}_{t+1} - \overline{R} &= \frac{\Omega}{(\Omega-1)} E_t \left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \frac{\Omega\theta}{(\Omega-1)} E_t \left(\overline{g}_{c,t+1} - \overline{g}\right) \\ &\quad - \frac{\Omega\psi(\theta-1)\frac{l}{1-l}}{(\Omega-1)} E_t \overline{g}_{l,t+1} - \frac{\Omega_V}{(\Omega-1)} E_t \overline{g}_{V,t} - \frac{1}{(\Omega-1)} \left(\overline{R}_t - \overline{R}\right). \end{aligned} \tag{30}$$

On the one hand, this compares better to standard interest rate rules which feature the lagged dependent variable on the right hand side as an 'interest rate smoothing' term but on the other hand, the lead nominal interest rate now stands as the dependent variable which is not usually the case.

Conceptually, this also raises the issue that McCallum (2010) identifies. He argues that the equilibrium conditions of a structural model stipulate whether any given difference equation is forward-looking ("expectational") or backward-looking ("inertial") and that the researcher is not free to alter the direction of causality implied by the model as is convenient. The forward looking representation of the Taylor condition (23) is the long accepted rational expectations version; for example, Lucas (1980) suggests that the forward looking "filters" suit models which feature an optimizing consumer.

6.2 Credit Interpretation of the Taylor Condition

Christiano et al. (2010) have considered how the growth rate of credit might be included as part of a Taylor rule:

> "Inflation is low during stock market booms, so that an interest rate rule that is too narrowly focused on inflation destabilizes asset markets and the broader economy. Adjustments to the interest rate rule can remove this source of welfare-reducing instability. For example, allowing an independent role for credit growth (beyond its role in constructing the inflation forecast) would reduce the volatility of output and asset prices."

The term on the growth of velocity can also be interpreted as a growth of credit in the following way:

$$\begin{aligned} V_t &= \frac{1}{1-\left(1-\frac{m_t}{c_t}\right)} = \frac{1}{\frac{m_t}{c_t}}, \\ V\widehat{V}_t &= \left(1-\frac{m}{c}\right)\widehat{\left(1-\frac{m_t}{c_t}\right)} \end{aligned}$$

so that:

$$\overline{g}_{V,t} = \frac{m}{c}\left(1-\frac{m}{c}\right)\overline{g}_{\left(1-\frac{m}{c}\right),t}$$

where $\overline{g}_{\left(1-\frac{m}{c}\right),t}$ is the growth rate of credit (per real money). The modified Taylor condition in the linear deviation form (with consumption growth) is

$$\begin{aligned} \overline{R}_t - \overline{R} &= \Omega E_t\left(\overline{\pi}_{t+1}-\overline{\pi}\right) + \Omega\theta E_t\left(\overline{g}_{c,t+1}-\overline{g}\right) + \Omega\psi\left(1-\theta\right)\frac{l}{1-l}E_t\overline{g}_{l,t+1} \\ &\quad -\Omega_{\left(1-\frac{m}{c}\right)}E_t\overline{g}_{\left(1-\frac{m}{c}\right),t+1} - \Omega_R E_t\left(\overline{R}_{t+1}-\overline{R}\right). \end{aligned} \tag{31}$$

Corollary 4 *The Taylor condition in equation (31) is characterized by* $\Omega_{\left(1-\frac{m}{c}\right)} \equiv \frac{(R-1)(1-\gamma-\omega)^2}{(1-\gamma)^2}\frac{\omega}{R(1-\omega)}$; *given* w, *then* $\frac{\partial\Omega_{\left(1-\frac{m}{c}\right)}}{\partial R} > 0$, $\frac{\partial\Omega_{\left(1-\frac{m}{c}\right)}}{\partial\gamma} > 0$.

A positive expected credit growth rate causes a negative effect on the current net nominal interest rate $\overline{R}_t$, which rises in magnitude as the *BGP* R rises and as the credit technology parameter γ rises.

6.3 Taylor Condition for a CIA Endogenous Growth Economy

One special case of the general Benk et al. (2010) model is where there is no banking and no credit production; a cash-only economy remains. This holds if $A_F = 0$ in the Benk et al. (2010) model and is equivalent to an endogenous growth model with a standard CIA constraint, as in Gomme (1993) or similarly Jones et al. (2007). In this case, the Taylor condition can be expressed as:

$$\overline{R}_t - \overline{R} = E_t\left(\overline{\pi}_{t+1}-\overline{\pi}\right) + \theta E_t\left(\overline{g}_{c,t+1}-\overline{g}\right) + \psi\left(1-\theta\right)E_t\overline{g}_{l,t+1}. \tag{32}$$

The most striking differences here are that the Taylor condition does not contain the terms in velocity growth and the forward interest rate term (the 'interest rate smoothing' terms) and the Taylor principle is only marginally met in that $\Omega = 1$, as opposed to $\Omega > 1$.

The difference between the Friedman (1969) optimum special case for equation (32) which implied $\Omega = 1$ and the same result for equation (23) is that in the case of the full model the credit service was optimally disregarded by consumers because money carries no opportunity cost when the (net) nominal

interest rate is equal to zero. Here it is simply not available and so $\Omega = 1$ always holds. Estimating equation (32) using data simulated from the cash-only, endogenous growth model would therefore be expected to yield an inflation coefficient of unity. In other words, the theoretical prediction would be for a Fisher relationship between the nominal interest rate and inflation, controlling for consumption growth and productive time growth.[19]

This derivation emphasises the point that velocity must be free to rise above unity in order for the coefficient on inflation to rise above one in this model and in order for the 'interest rate smoothing terms' to appear in the Taylor condition. The Benk et al. (2010) model achieves this by modelling an endogenous cash-credit choice.

6.4 Taylor Condition with No Physical Capital

Retaining the endogenous velocity component of the Benk at al. (2010) model ($A_F > 0$) but specifying an economy without physical capital results in a human capital only economy, as in the framework of Gillman and Kejak (2005) or similarly Hromcová (2008). The following Taylor condition is obtained:

$$\begin{aligned} \overline{R}_t - \overline{R} &= \Omega E_t \left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega\theta E_t \left(\overline{g}_{y,t+1} - \overline{g}\right) + \Omega\psi \left(1-\theta\right) \frac{l}{1-l} E_t \overline{g}_{l,t+1} \\ &\quad -\Omega_V E_t \overline{g}_{V,t+1} - \Omega_R E_t \left(\overline{R}_{t+1} - \overline{R}\right). \end{aligned} \tag{33}$$

Where the consumption growth rate coincides with the output growth rate but the Taylor condition still features terms in velocity growth and the forward interest rate term. The Taylor principle is also still satisfied ($\Omega > 1$) away from the Friedman optimum nominal interest rate ($R > 1$). If we were to simulate data from this restricted version of the Benk et al. (2010) model then equation (28), which was previously misspecified in the full version of the model, would now be the appropriate estimating equation to apply to the simulated data.

Consider the case where there is no leisure choice ($\psi = 0$), or alternatively where the utility function takes a logarithmic form ($\theta = 1$), then the Taylor condition no longer includes the term for the expected growth in productive time:

$$\begin{aligned} \overline{R}_t - \overline{R} &= \Omega E_t \left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega\theta E_t \left(\overline{g}_{y,t+1} - \overline{g}\right) \\ &\quad -\Omega_V E_t \overline{g}_{V,t+1} - \Omega_R E_t \left(\overline{R}_{t+1} - \overline{R}\right). \end{aligned}$$

And if there is no banking ($A_F = 0$) then the Taylor condition collapses to a forward-looking variant of Taylor's (1993) rule without 'smoothing' terms:

$$\overline{R}_t - \overline{R} = E_t \left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \theta E_t \left(\overline{g}_{y,t+1} - \overline{g}\right). \tag{34}$$

[19] Arnwine and Yigit (2008) consider an an "augmented Fisher relation" which features a one-for-one relationship between the nominal interest rate and inflation, controlling for consumption growth.

The appropriate estimating equation to apply to data simulated from this limited version of the general model would be equation (29) under the restriction that $\beta_5 = 0$. However, an inflation coefficient in excess of unity is not admissible according to the Taylor condition derived under these circumstances and we revert back to a one-to-one relationship between inflation and the nominal interest rate; a Fisher relationship (controlling for output growth this time).

6.5 Taylor Condition for an Exogenous Growth Economy

Interestingly, the GE Taylor rule has the same form for an exogenous growth variant of the general model:

$$\begin{aligned}\overline{R}_t - \overline{R} &= \Omega E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega\theta E_t\left(\overline{g}_{c,t+1} - \overline{g}\right) + \Omega\psi\left(1-\theta\right)\frac{l}{1-l}E_t\overline{g}_{l,t+1} \\ &\quad -\Omega_V E_t\overline{g}_{V,t+1} - \Omega_R E_t\left(\overline{R}_{t+1} - \overline{R}\right).\end{aligned}$$

Where $\overline{g}$ now denotes the exogenous growth rate of labor-augmenting productivity. And if there is no physical capital, then again $y = c$ and:

$$\begin{aligned}\overline{R}_t - \overline{R} &= \Omega E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega\theta E_t\left(\overline{g}_{y,t+1} - \overline{g}\right) + \Omega\psi\left(1-\theta\right)\frac{l}{1-l}E_t\overline{g}_{l,t+1} \\ &\quad -\Omega_V E_t\overline{g}_{V,t+1} - \Omega_R E_t\left(\overline{R}_{t+1} - \overline{R}\right).\end{aligned}$$

With no leisure preference, and no banking ($A_F = 0$), again the simple standard rule results:

$$\overline{R}_t - \overline{R} = E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \theta E_t\left(\overline{g}_{y,t+1} - \overline{g}\right).$$

Despite the fact that the above Taylor conditions look identical for the models with exogenous and endogenous growth, there is a fundamental difference between the expressions derived from each class of model. Most importantly, under the exogenous growth setup the targeted inflation rate and growth rate of the economy are completely unrelated (and so considered independently). This could be compared to a conventional interest rate rule in which the inflation target is assumed to be constant and potential output is assumed to grow at a rate determined by a statistical trend. In contrast, under the endogenous growth setup the targets for inflation, the growth rate, and the nominal interest rate are all endogenously determined. Only in the log-utility case is the nominal interest R independent of the growth rate, and a simple additive function of (approximately) the stationary money supply growth rate and the rate of time preference. However, in the log-utility case, the BGP inflation rate and growth rate are still simultaneously determined as endogenous variables.

7 Discussion

Expressing the monetary policy process in terms of the nominal interest rate has the advantage of reconciling the language of economists who have traditionally depicted the money supply as the instrument of monetary policy with

the language of central bankers, who are more accustomed to conducting policy deliberations in terms of a short-term interest rate (Mehrling, 2006). Alvarez et al. (2001) caution that modelling monetary policy solely in terms of a nominal interest rate rejects the quantity theory in spite of the strong empirical link between money growth, inflation and interest rates. Schabert (2003), for example, uses the equilibrium conditions of a standard cash-in-advance (CIA) model in order to derive the conditions under which a money supply rule and an interest rate rule are 'equivalent', while Fève and Auray (2002) generate simulated data from a similar model and demonstrate that an interest rate rule can be spuriously recovered from this data even though monetary policy is modelled in terms of a money growth rule.

This paper has derived an expression similar to a conventional interest rate rule as an equilibrium condition of an endogenous growth model with endogenous velocity in which monetary policy is characterized as a stochastic money supply rule. The theoretical model underpinning this expression implies that the coefficient on inflation exceeds unity in general, takes a value of unity as a special case at the Friedman (1969) optimum but that it may not fall below unity. Simulation exercises support the theoretical restriction placed on this coefficient, so long as the estimating equation accurately reflects the structure of the equilibrium condition.

Our results can be interpreted in several ways. First, the derivation could be said to represent an 'equivalence proposition' between the money supply process modelled and the 'interest rate rule' which actually represents an equilibrium condition of the model. This would be similar to the interpretation adopted by studies such as Végh (2002) and Schabert (2003).

Second, the Taylor condition can be interpreted as the interest rate rule which results from the money supply process in the context of the Benk et al. (2010) model. Woodford similarly derives the interest rate rule which "implements" strict inflation targeting in the New Keynesian model (Woodford, 2003, pp.290-295). However, the money supply does not enter that model. Changes in the velocity of money therefore play no role and thus cannot be used to help explain why traditional Taylor rule estimations might use misspecified estimating equations in order to obtain an inflation coefficient of less than one. The fact that our framework assigns a central role to money (as in Alvarez et al., 2001) potentially implies that the money growth rule can offer guidance to policymakers at times when the conventional monetary policy instrument encounters the zero lower bound, as is the case at the present time.

Third, the 'Taylor condition' derived in this paper could be interpreted as adding to the intrigue of what empirical studies actually find when they recover interest rate rules from time series data. Hetzel (2000), for example, warns that empirical correlations between a short-term interest rate and macroeconomic variables such as output and inflation cannot be interpreted to reveal the behaviour of policymakers (i.e. their policy rule) unless the relationship obtained can be declared as structural. Cochrane (2011) also argues that the Taylor rule suffers from an identification problem in the New Keynesian model. Clearly a reduced form expression estimated from the data can be consistent with a

variety of potential explanations. Our contribution has been to offer one very particular explanation based on a neoclassical monetary model extended to include endogenous growth and endogenous velocity in order to shed light on the structural relationships which might underpin the reduced form expressions to which Hetzel (2000) refers.

8 Conclusion

The paper has derived a general equilibrium condition in dynamic and balanced growth path cases for a constant relative risk aversion economy with leisure, endogenous growth through a separate human capital investment sector as in Lucas (1988), and with endogenous velocity through production of exchange credit in a financial intermediary as grounded in that literature. The importance of velocity in reproducing the 'Taylor principle' is consistent with the important role for velocity reported by Reynard (2004). The paper also looked at special cases with log-utility and with a simple cash-in-advance constraint in which velocity is one. While providing a theoretical means to overview the Taylor empirical literature, such as reviewed by Siklos and Wohar (2005), here the immediate focus is first to show that estimation of a Taylor rule may result in a spurious inference that the central bank is engaged in Taylor principle behaviour, rather than simply supplying money. This is established here by generating artificial data as simulated from the model and then estimating successfully our theoretical Taylor condition. This condition is simply an equilibrium condition in the economy in which the central bank stochastically makes changes in the money supply growth rate to finance government spending. For example, such money supply changes tend to occur whenever the government needs to resort to the 'fiscal inflation tax', as in the aftermath of the current banking crisis and recession, or during war in general, which some construe to be the source of the deficit problem of the last decade.

Money velocity growth itself enters as a variable and ends up playing a potentially significant role; in particular when velocity is changing, such as during the recent banking crisis and during the 1930s when velocity cycled downwards, as identified in Benk et al. (2010). Velocity is endogenised in the model following the banking financial intermediation microeconomic literature of producing financial services with a Cobb-Douglas production function that includes deposited funds as an input; this approach implies a bank service sector value-added that is consistent with the US national income accounting treatment of the bank service sector.

The paper exhibits how the banking production of exchange credit is surprising crucial to the derivation of a Taylor principle whereby the coefficient on the inflation term is in fact greater than one. This results only through an endogenous velocity of money; a simple CIA (cash-only) constraint with a constant velocity of one is shown to provide an inflation coefficient of unity. Through endogenous growth, we can derive an output gap measure not inconsistent with Taylor and Wieland's (2010) emphasis on changes in output as a measure for

the output gap. In our model, the output growth term does enter directly but only if we also include an investment growth term; otherwise the consumption growth is the 'output gap' term of the model's Taylor condition.

Estimation results are also given for certain misspecified models, relative to the generated data, and indeed the effects on the results are substantial. The results may give a way to better understand how to estimate Taylor rules so as to better account for changes in underlying conditions, including times when changes in velocity are significant because of bank productivity collapses. Omitting this term, as almost all conventional Taylor rules do, may induce significant omitted variable bias. Therefore the results hold promise for explaining comparisons of estimated rules across different periods and countries, as well as during bank crises, sudden financial deregulation, or times of other significant shifts in money velocity, a task which would help organize this disparate literature.

By simulating data of the model and estimating successfully a 'Taylor rule' from the data, the paper implies that identification of such a rule econometrically can be achieved as part of the economy's asset pricing behaviour when the central bank simply prints money stochastically. In that case it would be spurious to claim that such Taylor estimations show how the central bank actually conducts policy through interest rate targeting rather than through simply satisfying its fiscal needs via direct and indirect taxes, including the inflation tax.

References

[1] Alvarez, Fernando, Robert E. Lucas Jr. and Warren E. Weber (2001), "Interest Rates and Inflation", American Economic Review, vol. 91(2), pp. 219-225, May.

[2] Arnwine, Neil and Taner M. Yigit (2008), "What Fisher Knew About His Relation, We Sometimes Forget", Economics Letters, vol. 101(3), pp. 193-195, December.

[3] Benk, Szilárd, Max Gillman, and Michal Kejak (2008), "Money Velocity in an Endogenous Growth Business Cycle with Credit Shocks", Journal of Money, Credit, and Banking, vol. 40(6), pp. 1281-1293, September.

[4] Benk, Szilárd, Max Gillman and Michal Kejak (2010), "A Banking Explanation of the US Velocity of Money: 1919-2004", Journal of Economic Dynamics and Control, vol. 34(4), pp. 765–779, April.

[5] Bernanke, Benjamin S. and Mark L. Gertler (2001), "Should Central Banks Respond to Movements in Asset Prices?", American Economic Review, vol. 91(2), pp. 253-257, May.

[6] Canzoneri, Matthew B. and Cumby, Robert E. and Diba, Behzad T., 2007. "Euler equations and money market interest rates: A challenge for monetary policy models," Journal of Monetary Economics, vol. 54(7), pages 1863-1881, October.

[7] Carare, Alina and Robert Tchaidze (2005), "The Use and Abuse of Taylor Rules: How Precisely Can We Estimate Them?", IMF Working Paper, 05/148, July.

[8] Carlstrom, Charles T. and Timothy S. Fuerst (2001), "Timing and Real Indeterminacy in Monetary Models", Journal of Monetary Economics, vol. 47(2), pp. 285-298, April.

[9] Cecchetti, Stephen G., Hans Genberg and Sushil Wadhwani (2000), "Asset Prices in a Flexible Inflation Targeting Framework", in: William C. Hunter, George G.Kaufman and Michael Pomerleano (eds.), "Asset Price Bubbles: Implications for Monetary, Regulatory, and International Policies", MIT Press, Cambridge, Mass. pp. 427-444.

[10] Chowdhury, Ibrahim and Andreas Schabert (2008), "Federal Reserve Policy Viewed Through a Money Supply Lens", Journal of Monetary Economics, vol. 55(4), pp. 825-834, May.

[11] Christiano, Lawrence J. and Terry J. Fitzgerald (2003), "The Band Pass Filter", International Economic Review, vol. 44(2), pp. 435-465, May.

[12] Christiano, Lawrence J., Cosmin L. Ilut, Roberto Motto and Massimo Rostagno (2010) "Monetary Policy and Stock Market Booms", NBER Working Papers 16402, September.

[13] Clarida, Richard, Jordi Galí and Mark L. Gertler (1998), "Monetary Policy Rules in Practice Some International Evidence", European Economic Review, vol. 42(6), pp. 1033-1067, June.

[14] Clarida, Richard, Jordi Galí and Mark L. Gertler (1999), "The Science of Monetary Policy: A New Keynesian Perspective", Journal of Economic Literature, vol. 37(4), pp. 1661-1707, December.

[15] Clarida, Richard, Jordi Galí and Mark L. Gertler (2000), "Monetary Policy Rules And Macroeconomic Stability: Evidence And Some Theory", The Quarterly Journal of Economics, vol. 115(1), pp. 147-180, February.

[16] Clark, Jeffrey A. (1984), "Estimation of Economies of Scale in Banking Using a Generalized Functional Form", Journal of Money, Credit and Banking, vol. 16(1), pp. 53–68, February.

[17] Cochrane John H. (2011), "Determinacy and Identification with Taylor Rules", Journal of Political Economy, vol. 119(3), pp. 565-615, June.

[18] Comin, Diego and Mark Gertler (2006), "Medium-Term Business Cycles", American Economic Review, vol. 96(3), pp. 523-551, June.

[19] Fève, Patrick and Stéphane Auray (2002), "Interest Rate and Inflation in Monetary Models with Exogenous Money Growth Rule", Economics Bulletin, vol. 5(1), pp. 1-10.

[20] Fountas, Stilianos, Menelaos Karanasos and Jinki Kim (2006), "Inflation Uncertainty, Output Growth Uncertainty and Macroeconoimc Perfomance", Oxford Bulletin of Economics and Statistics, vol. 68(3), pp. 319-343, June.

[21] Friedman, Milton (1969), "The Optimum Quantity of Money", in: "The Optimum Quantity of Money and Other Essays", Chicago: Aldine.

[22] Gillman, Max, Mark Harris and László Mátyás (2004), "Inflation and Growth: Explaining the Negative Effect", Empirical Economics, vol. 29(1), pp. 149-167, January.

[23] Gillman, Max and Michal Kejak (2005), "Inflation and Balanced-Path Growth with Alternative Payment Mechanisms", Economic Journal, vol. 115(500), pp. 247-270, January.

[24] Gillman, Max and Michal Kejak (2011), "Inflation, Investment and Growth: a Banking Approach", Economica, vol. 78(310), pp. 260-282, April.

[25] Gomme, Paul (1993), "Money and Growth Revisited: Measuring the Costs of Inflation in an Endogenous Growth Model", Journal of Monetary Economics, vol. 32(1), pp. 51-77, August.

[26] Haug, Alfred A. and William G. Dewald (2011), "Money, Output, and Inflation in the Longer Term: Major Industrial Countries, 1880-2001", Economic Inquiry, forthcoming.

[27] Hetzel, Robert L. (2000), "The Taylor Rule: Is It a Useful Guide to Understanding Monetary Policy?", Federal Reserve Bank of Richmond Economic Quarterly, vol. 86/2, Spring.

[28] Hromcová, Jana (2008), "Learning-Or-Doing in a Cash-In-Advance Economy with Costly Credit", Journal of Economic Dynamics and Control, vol. 32(9), pp. 2826-2853, September.

[29] Ireland, Peter N. (2007), "Changes in the Federal Reserve's Inflation Target: Causes and Consequences", Journal of Money, Credit and Banking, vol. 39(8), pp. 1851-1882, December.

[30] Jondeau, Eric, Herve Le Bihan and Clementine Galles (2004), "Assessing Generalized Method-of-Moments Estimates of the Federal Reserve Reaction Function", Journal of Business & Economic Statistics, vol. 22, pp. 225-239, April.

[31] Jones, Larry E., Rodolfo E. Manuelli, and Henry E. Siu (2005), "Fluctuations in Convex Models of Endogenous Growth II: Business Cycle Properties", Review of Economic Dynamics, vol. 8(4), pp. 805-828, October.

[32] Lucas, Robert Jr, 1976. "Econometric policy evaluation: A critique," Carnegie-Rochester Conference Series on Public Policy, vol. 1(1), pages 19-46, January.

[33] Lucas, Robert E. Jr. (1980), "Two Illustrations of the Quantity Theory of Money", American Economic Review, vol. 70(5), pp. 1005-14, December.

[34] Lucas, Robert E. Jr. (1988), "On the Mechanics of Economic Development", Journal of Monetary Economics, vol. 22(1), pp. 3-42, July.

[35] Mankiw, N. Gregory (2001), "U.S. Monetary Policy During the 1990s", NBER Working Papers 8471, September.

[36] McCallum, Bennett T. and Edward Nelson (1999), "Performance of Operational Policy Rules in an Estimated Semiclassical Structural Model", NBER Chapters, in: John B. Taylor (ed.), "Monetary Policy Rules", pp. 15-56.

[37] McCallum, Bennett T. (2010), "Indeterminacy, Causality, and the Foundations of Monetary Policy Analysis", Swiss Journal of Economics and Statistics, vol. 146(I), pp. 107-120, March.

[38] Mehrling, Perry "Mr. Woodford and the Challenge of Finance", Journal of the History of Economic Thought, vol. 28(2), pp. 161-170, June.

[39] Orphanides, Athanasios (2001), "Monetary Policy Rules Based on Real-Time Data", American Economic Review, vol. 91(4), pp. 964-985, September.

[40] Orphanides, Athanasios (2008), "Taylor Rules", in: Steven N. Durlauf and Lawrence E. Blume (eds.), "The New Palgrave Dictionary of Economics", Second Edition.

[41] Poole, William (1970), "Optimal Choice of Monetary Policy Instruments in a Simple Stochastic Macro Model", The Quarterly Journal of Economics, vol. 84(2), pp. 197-216, May.

[42] Ravn, Morten O. and Harald Uhlig (2002), "On Adjusting the Hodrick-Prescott Filter for the Frequency of Observations", The Review of Economics and Statistics, vol. 84(2), pp. 371-375, May.

[43] Reynard, Samuel (2004), "Financial Market Participation and the Apparent Instability of Money Demand", Journal of Monetary Economics, vol. 51(6), pp. 1297-1317, September.

[44] Rudebusch, Glenn D. (2009), "The Fed's Monetary Policy Response to the Current Crisis", FRBSF Economic Letter, Federal Reserve Bank of San Francisco, Issue: May.

[45] Salyer, Kevin and Kristin Van Gaasbeck (2007), "Taking the Monetary Implications of a Monetary Model Seriously", Economics Bulletin, vol. 5(20), pp. 1-7.

[46] Sargent, Thomas J. and Paolo Surico (2008), "Monetary Policies and Low-Frequency Manifestations of the Quantity Theory", Discussion Papers 26, Monetary Policy Committee Unit, Bank of England, December.

[47] Sargent, Thomas J. and Paolo Surico (2011), "Two Illustrations of the Quantity Theory of Money: Breakdowns and Revivals", American Economic Review, vol. 101(1), pp. 109-128, February.

[48] Schabert, Andreas (2003), "On the Equivalence of Money Growth and Interest Rate Policy", working paper 2003-6, Department of Economics, University of Glasgow, April.

[49] Siklos, Pierre L., and Mark E. Wohar (2005), "Estimating Taylor Type Rules: An Unbalaned Regression?", in: Thomas B. Fomby and Dek Terrell (eds.) "Advances in Econometrics", vol. 20, Amsterdam: Elsevier.

[50] Sørensen, Peter Birch and Hans Jorgen Whitta-Jacobsen (2005), "Introducing Advanced Macroeconomics: Growth and Business Cycles", McGraw-Hill.

[51] Taylor, John, B. (1993), "Discretion versus Policy Rules in Practice", Carnegie-Rochester Conference Series on Public Policy, vol. 39(1), pp. 195-214.

[52] Taylor, John B. (1999), "A Historical Analysis of Monetary Policy Rules", NBER Chapters, in: John B. Taylor (ed.), " Monetary Policy Rules", pp. 319-347.

[53] Taylor, John, B. and Volker Wieland (2010), "Surprising Comparative Properties of Monetary Models: Results from a New Model Database", ECB Working Paper 1261, November.

[54] Végh, Carlos (2002), "Monetary Policy, Interest Rate Rules, and Inflation Targeting: Some Basic Equivalences", Central Banking, Analysis, and Economic Policies Book Series, in: Fernando Lefort & Klaus Schmidt-Hebbel and Norman Loayza (Series Editor) and Klaus Schmidt-Hebbel (ed.), "Indexation, Inflation and Monetary Policy", vol. 2, ch. 6, pp. 151-182.

[55] Walsh, Carl E. (2003), "Speed Limit Policies: The Output Gap and Optimal Monetary Policy", American Economic Review, vol. 93(1), pp. 265-278, March.

[56] Woodford, Michael (2003), "Interest and Prices: Foundations of a Theory of Monetary Policy", Princeton University Press.

[57] Woodford, Michael (2008), "How Important Is Money in the Conduct of Monetary Policy?", Journal of Money, Credit and Banking, vol. 40(8), pp. 1561-1598, December.