# Autocorrelated Returns, Time-Varying Higher Distributional Moments and the Cumulative Prospect Theory

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**Abstract**

Trading mechanisms, time-varying risk factors and the reaction speed to new market information, among other things, promote autocorrelated returns and time-varying higher distributional moments of investment opportunities. This study provides a bootstrap approach to empirically implementing the cumulative prospect theory (CPT) while considering autocorrelation and allowing for time-varying higher distributional moments. An empirical analysis reevaluates the effect of myopic loss aversion and shows that the market portfolio gains significantly in attractiveness against the risk-free asset when applying the proposed approach. This gain in attractiveness is mostly attributed to the time variation of higher distributional moments of the market portfolio. Nevertheless, evidence demonstrates that more pronounced serial correlation patterns in the return series also affect the attractiveness of the market portfolio. Hence, the approach of this study should be applied when, for example, evaluating investment opportunities, which are based on time series data using the CPT.

**Keywords:** Return autocorrelation; time-varying higher distributional moments; cumulative prospect theory; autoregressive sieve bootstrap; myopic loss aversion

**JEL Classifications:** C15; G02; G12

## 1. Introduction

Stock returns usually exhibit autocorrelation patterns (see, for example, Poterba & Summers (1988)) or time-varying higher distributional moments (see, for example, Bollerslev (1986), Harvey & Siddique (1999) and Brooks et al. (2005)). Benartzi & Thaler (1995), Durand et al. (2004), Zeisberger et al. (2007), Dierkes et al. (2010) and Fong (2013) propose bootstrap approaches for empirically implementing the cumulative prospect theory (CPT) given a time series of return observations. These approaches either do not allow the consideration of autocorrelation in the underlying return series, make subjective and thus unreliable assumptions on the serial correlation, or do not allow higher distributional moments to vary over time. These three facts raise two issues. First, results are doubtful when applying several of these approaches to an autocorrelated return series because Singh (1981) and Babu & Singh (1983) found that a bootstrap, which makes erroneous assumptions on the autocorrelation of the underlying time series, leads to improper convergence of the moments of the bootstrapped statistic. Second, CPT values are likely to be time-varying because CPT investors are sensitive to higher distributional moments, which are also time-varying. Ågren (2006a) provides evidence that considering for example conditional variance affects CPT value estimates whereas Ågren (2006b), Barberis & Huang (2008) and Gregory-Allen et al. (2010) show the relationship between CPT values and distributional characteristics such as variance, skewness or kurtosis.

This study therefore aims to identify whether considering autocorrelation in underlying return series and time-variation of higher distributional moments affects the CPT value estimates and their inference statistics. As such, this study uses an autoregressive (AR) sieve bootstrap approach to compute CPT values and their inference statistics for multi-period investment horizons. Furthermore, an empirical study compares the procedure of Zeisberger et al. (2007) with the proposed approach by reevaluating the effect of myopic loss aversion (see, for example, Benartzi & Thaler (1995), Gneezy et al. (2003) and Haigh & List (2005)). This effect describes the dependence of the risk perception of CPT investors on the investment horizon and provides a possible explanation for the famous equity premium puzzle (see, for example, Mehra & Prescott (1985)).

The empirical analysis of this study shows that the monthly return series of the market portfolio bears weak but significant short-term persistence. Furthermore, statistical tests show that the empirical second to the fourth central moments of the return distribution of the market portfolio are significantly time-varying. Therefore, assessing the attractiveness of the market portfolio against the risk-free asset leads to higher CPT values for the former when the approach of this study is applied. This additional gain in attractiveness is mainly attributed to the consideration of time-varying higher distributional moments. Nevertheless, evidence is provided that shows that more pronounced patterns of temporal dependence in the underlying return series also significantly affect CPT value estimates and their inference statistics. Consequently, both approaches still support the effect of myopic loss aversion. However, when applying the approach proposed in this study, the risky market portfolio is less attractive at a 95% significance level than the risk-free asset for investment horizons of 1 to 3 months and equally attractive for lower evaluation frequencies, against which the procedure of

Zeisberger et al. (2007) implies a lower attractiveness for investment horizons of 1 to 6 months.

The remainder of this paper is laid out as follows: Section 2 introduces the CPT while Section 3 provides the AR-sieve bootstrap approach for empirically implementing the CPT. Section 4 compares the approach of Zeisberger et al. (2007) with the algorithm proposed in this study. Section 5 concludes with a summary of the study's important findings.

## 2. Overview of the Cumulative Prospect Theory

Tversky & Kahneman (1992) developed the CPT by combining the rank-dependent utility theory developed by Quiggin (1982) with the prospect theory developed by Kahneman & Tversky (1979).

Let $(x_{-m}, p_{-m}; \dots; x_{-1}, p_{-1}; x_0, p_0; x_1, p_1; \dots; x_n, p_n)$ be a lottery $L$ with outcomes $x_i$ and corresponding probabilities $p_i$. Furthermore, $x_i < x_j$ for $i < j$ and $x_0 = 0$. Tversky and Kahneman (1992) define the CPT value $V$ of a lottery $L$ as follows:

$$V(L) = \sum_{i=-m}^{n} \pi(p_i) \cdot v(x_i) = \sum_{i=-m}^{0} \pi^-(p_i) \cdot v^-(x_i) + \sum_{i=0}^{n} \pi^+(p_i) \cdot v^+(x_i) \tag{1}$$

Here, $v^-(x_i)$ and $v^+(x_i)$ are strictly increasing convex or concave functions that return values for the losses and gains of $L$, respectively. Let $T^-(P)$ and $T^+(P)$ be two strictly increasing functions that transform the cumulative probabilities $\sum p = P$ of these losses and gains. Next, define $\pi^-(p_i)$ and $\pi^+(p_i)$, respectively, as follows:

$$\pi^-(p_i) = T^-(p_{-m} + \dots + p_i) - T^-(p_{-m} + \dots + p_{i-1}) \tag{2}$$

$$\pi^+(p_i) = T^+(p_i + \dots + p_n) - T^+(p_{i+1} + \dots + p_n) \tag{3}$$

The operations in (2) and (3) are adopted from rank-dependent utility theory. Tversky & Kahneman (1992) define $T^-(P)$ and $T^+(P)$, respectively, as follows:

$$T^-(P) = \frac{p^{\gamma^-}}{(p^{\gamma^-} + (1-p)^{\gamma^-})^{\frac{1}{\gamma^-}}} \tag{4}$$

$$T^+(P) = \frac{p^{\gamma^+}}{\left(p^{\gamma^+} + (1-p)^{\gamma^+}\right)^{\frac{1}{\gamma^+}}} \tag{5}$$

Using survey data, Tversky & Kahneman (1992) estimate $\gamma^-$ and $\gamma^+$ to be 0.69 and 0.61, respectively. The estimates and functional forms of the transformation functions then lead to an overestimation of low probabilities and an underestimation of high probabilities.

For the value functions, Tversky & Kahneman (1992) propose an unbounded power function with a kink at the origin. Therefore, $v^-(x_i)$ and $v^+(x_i)$ are, respectively, defined as follows:

$$v^-(x_i) = -\lambda \cdot (-x_i)^{\alpha^-} \tag{6}$$

$$v^+(x_i) = x_i^{\alpha^+} \tag{7}$$

Tversky & Kahneman (1992) estimate $\alpha^-$ and $\alpha^+$ equally at 0.88 and the loss aversion $\lambda$ at 2.25. Therefore, investors will be risk-seeking for losses and risk-averse for gains when evaluating lotteries with (6) and (7). Furthermore, a loss aversion of 2.25 implies that losses cost 2.25 times more in disutility than is gained in spending utility.

## 3. Implementing the Cumulative Prospect Theory

Estimating CPT values as in (1) together with their inference statistics for investment opportunities at different evaluation frequencies *n* based on time series data requires *n*-month return distributions. The latter can be obtained empirically by applying a bootstrap to the underlying return series. A bootstrap approach should then correctly consider such characteristics as return autocorrelation or time-varying higher distributional moments.

Applying a bootstrap with erroneous assumptions on the temporal dependence structure of the underlying return series will likely lead to improper convergence of the estimated moments of the distribution of the bootstrapped statistics, as discussed, for example, by Singh (1981) and Babu & Singh (1983). Here, the bootstrapped statistic is the *n*-month return. The resulting *n*-month return distribution is the basis for the CPT value estimates and thus, improper distributional moments might affect CPT value estimates and in turn their inference statistics.

Furthermore, allowing for time-varying higher distributional moments, which are discussed, for example in Bollerslev (1986), Harvey & Siddique (1999) and Brooks et al. (2005), also affects the results. Consequently, when the underlying *n*-month return distributions vary over time it is likely that the CPT values, which are based on these distributions, are also varying. The latter relationship is explained by the sensitivity of the CPT investors towards the higher moments of a return distribution. This sensitivity is attributed to the probability weighting functions in (4) and (5) or to the asymmetric value functions in (6) and (7) as discussed, for example, in Ågren (2006b), Barberis & Huang (2008) and Gregory-Allen et al. (2010).

Benartzi & Thaler (1995) propose a first bootstrap approach to compute CPT values on the basis of time series data. From the entire sample of monthly portfolio returns, they draw with replacement *n* single observations and compute the *n*-month portfolio return. They repeat these steps 100,000 times to approximate the portfolio return distribution for an *n*-month investment horizon. This bootstrap approach assumes independently and identically distributed returns and does not allow the computation of inference statistics for the estimated CPT values. The assumption of independence is not considered very serious by Benartzi & Thaler (1995). They argue that the CPT value estimates are not affected by the non-consideration of possible autocorrelation because the existing temporal dependence seems unsubstantial, which is supported in this study by a monthly frequency of the market portfolio returns. Instead, it is argued here that time-varying distributional moments affect the results substantially. Furthermore, the approach of Benartzi & Thaler (1995) cannot be applied to time series where autocorrelation is substantial.

Another approach is put forward by Durand et al. (2004). However, their method and results have been questioned by Zeisberger et al. (2007) because the approach of Durand et al. (2004) appears to suffer from miscomputations and a non-transparent description of the method.

Nevertheless, Durand et al. (2004) appear to use non-overlapping blocks of monthly returns to compute the *n*-month return distribution. In this way, Durand et al. (2004) obtain a sample of independent CPT values, which is a necessary requirement for their computation of inference statistics. However, the underlying return series must be sufficiently large to compute reliable inference statistics, which is problematic, particularly for long investment horizons. Nevertheless, this data structuring allows the return distributions to vary across blocks, although return autocorrelation inside individual blocks is still not considered.

Zeisberger et al. (2007) sample the original return series 5,000 times by drawing with replacement and subsequently applying the procedure of Benartzi & Thaler (1995), as described above, on each of the resampled series to obtain 5,000 CPT values. The latter distribution becomes the basis for computing inference statistics as, for example, in Trede (2002). To obtain a point estimate of the CPT value for an *n*-month evaluation frequency, Zeisberger et al. (2007) once again apply the method developed by Benartzi & Thaler (1995) to the entire original return series using 1,000,000 repetitions. The approach of Zeisberger et al. (2007) allows the computation of inference statistics for the point estimates of the CPT values but ultimately, their approach is just a repeated application of the procedure of Benartzi & Thaler (1995) and thus leads to the same critique. Zeisberger et al. (2007) also report a modification where they draw returns jointly from two time series, but this does not affect the results.

In contrast, Dierkes et al. (2010) and Fong (2013) apply a moving block bootstrap, as in Kunsch (1989), to return series. Both studies subjectively choose, on the one hand, the block lengths, but on the other hand, they justify their choices with economic reasons such as momentum effects. Fong (2013), for example, draws with replacement overlapping blocks of 60 monthly returns and chains the returns of four of these blocks to obtain a 20-month return. Nevertheless, neither study considers block length estimation procedures such as in Politis & White (2004). Applying this estimation procedure to the same time series of the approximated market portfolio Fong (2013) used leads to a block length of 3 monthly returns in lieu of the 60 chosen monthly returns. Furthermore, the approaches of Dierkes et al. (2010) and Fong (2013) do not explicitly allow for time-varying higher distributional moments and inference statistics cannot be computed for the CPT value estimates.

The above procedures are based on non-parametric bootstraps. In contrast, Ågren (2006a) proposes a model-based approach in which he estimates and simulates a generalized autoregressive conditional heteroscedasticity process. Consequently, Ågren (2006a) explicitly models the time-variation of the variance. Furthermore, he provides no inference statistics and focuses on the tail weights of a return distribution, neglecting, for example, possible conditional asymmetries or autoregressive mean processes of the time series. Nevertheless, Ågren (2006a) demonstrates that the bootstrap approach of Benartzi & Thaler (1995) does not correctly depict the distributional properties of the underlying return series and thus, neither does the allied approach of Zeisberger et al. (2007).

This study provides an approach to computing CPT values for multi-period investment horizons, which considers return autocorrelation and allows for time-varying higher distributional moments. This approach is based on a rolling *n*-month window consisting of

daily returns where the latter are chained to obtain *n*-month returns. Possible autocorrelation in the daily returns in each window is considered with the help of the AR-sieve bootstrap. The latter is, for example, discussed in Bühlmann (1997). Therefore, the serial correlation in monthly returns is bypassed by switching from a monthly return frequency to a daily return frequency to approximate the *n*-month return distributions.

Let $(R_t)_{t\in\mathbb{Z}}$ be a strictly stationary return data-generating process of length $T$, $\mu_R$ be the expected value of $R_t$ and $(\varepsilon_t)_{t\in\mathbb{Z}}$ be an innovation sequence of random variables, which are independently and identically distributed with zero mean. The AR-sieve bootstrap is then constructed with an AR-model of conditional order $p(T)$ as follows:

$$R_t - \mu_R = \sum_{i=1}^{p(T)} \phi_i \cdot (R_{t-i} - \mu_R) + \varepsilon_t \tag{8}$$

Kreiss et al. (2011) suggest using Yule-Walker estimators for the AR-parameters $\phi_i$ in (8) to guarantee that the bootstrapped time series is also stationary. The idea of the AR-sieve bootstrap is to approximate a time series with a sequence of finite-dimensional AR-processes of order $p(T)$ where $\lim_{T\to\infty} p(T) = \infty$ holds. Bühlmann (1997) determines $p(T)$ using the information criterion of Akaike (1974), which leads to an asymptotically efficient choice $\hat{p}$ of $p(T)$. Consequently, if $\hat{p} = 0$ holds, single return observations are drawn with replacement from the original time series, against which, if $\hat{p} > 0$ holds, (8) is fitted to the underlying return series. The whole time series is then rebuilt according to (8) based on the resampled centered residuals, which are computed from $T - \hat{p}$ estimates of $\varepsilon_t$.

Consequently, let $t_F^i$ and $t_L^i$ be the first and last trading day, respectively, of one of the $i = 1, \dots, T - (n-1)$ rolling windows consisting of $m_i$ daily returns, and let $\hat{V}_n^i$ be the estimate of the CPT value for the *i*th *n*-month return distribution of an investment opportunity. The AR-sieve bootstrap algorithm (ASBA) can be described as follows:

1. Choose $t_F^i$ and $t_L^i$ as the first and last trading day, respectively, of the *i*th *n*-month window.
2. Determine $\hat{p}$ for $m_i$ daily returns of the *i*th *n*-month window. If $\hat{p} = 0$, proceed with step 4. If $\hat{p} > 0$, estimate the specified AR-model and proceed with step 3.
3. Draw with replacement centered residuals from the fitted AR-model from step 2. Rebuild the original time series of daily returns of the *i*th *n*-month window according to the fitted AR-model and proceed with step 5.
4. Resample the original time series by drawing with replacement $m_i$ daily returns of the *i*th *n*-month window.
5. Compute the *n*-month portfolio return from the simulated time series by chaining the resampled $m_i$ daily returns of step 3 or step 4 for the *i*th *n*-month window.
6. Repeat steps 1 through 4 B times and compute $\hat{V}_n^i$ from the resulting empirical *n*-month return distribution for the *i*th *n*-month window, as in (1).

Applying the above algorithm leads to the time series $\hat{V}_n^i$ of $T - (n-1)$ CPT values for $i = 1, \dots, T - (n-1)$. This time series represents the distribution of $\hat{V}_n$, which denotes the overall CPT value of an investment opportunity. The time series $\hat{V}_n^i$ is assumed to better depict the distributional properties of the underlying *n*-month return series than, for example,

the approaches of Benartzi & Thaler (1995) or Zeisberger et al. (2007) because step 1 of the above algorithm segments the return series into $T-(n-1)$ periods, which allows the $n$-month return distribution to vary across the rolling windows. Consequently, the ASBA assesses the attractiveness of investment opportunities with respect to time-varying higher distributional moments. Possible autocorrelation in the underlying daily returns is addressed through steps 2 and 3.

The time series $\hat{V}_n^i$ can now be averaged to obtain a point estimate $\hat{V}_n$ of the overall CPT value of an investment opportunity. A general weighting formula for $\hat{V}_n$ with weight $w_i$ is defined as follows:

$$\hat{V}_n = \frac{\sum_{i=1}^{T-(n-1)} w_i \cdot \hat{V}_n^i}{\sum_{i=1}^{T-(n-1)} w_i} \tag{9}$$

When, for example, $w_i = 1$ holds for all $i$, (9) reduces to the simple arithmetic mean. $\hat{V}_n$ is then the average utility of a CPT investor who has an $n$-month evaluation frequency and who weights equally positive and negative events of past investment periods expressed by $\hat{V}_n^i$. One useful feature of this estimator is that other weighting schemes can be implemented in a straightforward manner. These schemes are discussed, for example, in Murdoch (1962), Kahneman et al. (1993) and Kemp et al. (2008).

Additionally, with the time series $\hat{V}_n^i$, it is straightforward to compute inference statistics for $\hat{V}_n$, based, for example, on robust standard errors, as per Andrews (1991). These robust standard errors become crucial when $n > 1$ holds because $\hat{V}_n^i$ is then calculated for rolling windows consisting of $n-1$ overlapping months of daily returns.

Nevertheless, there are some drawbacks to the ASBA. For example, $T$ must be sufficiently large to evaluate an investment opportunity of a CPT investor with a long investment horizon. If, for example, $T = n$ holds, then $\hat{V}_n^i$ consists of a single CPT value. Thus, $\hat{V}_n = \hat{V}_n^i$. Consequently, the ASBA does not allow the computation of inference statistics for $\hat{V}_n$ due to an insufficient number of observations of $\hat{V}_n^i$.

## 4. Myopic Loss Aversion Reconsidered

Benartzi & Thaler (1995) identify the effect of myopic loss aversion. This effect describes the dependency of the risk perception of CPT investors on their investment horizon. Benartzi & Thaler (1995) observed that the market portfolio is less attractive than the risk-free asset for evaluation frequencies of less than 12 months regarding the difference of the respective CPT values. This observation provides a possible explanation for the equity premium puzzle because investors demand higher risk premiums for shorter investment horizons than predicted by rational asset pricing theory. Furthermore, Zeisberger et al. (2007) provide inference statistics for the differences of these CPT values.

Following Barberis & Huang (2008), the empirical analysis of this study focuses on the estimated CPT value of the $n$-month excess return of the market portfolio $\hat{V}_n(R_M^e)$ instead of the difference in the CPT values of the market portfolio and the risk-free asset. Consequently,

the null hypothesis $\hat{V}_n(R_M^e) = 0$ provides evidence of whether the effect of myopic loss aversion exists and whether the market portfolio is in equilibrium for a specific investment horizon $n$. In addition, the empirical analysis compares the results of the ASBA and the approach of Zeisberger et al. (2007) because the latter is well documented and allows both the estimation of CPT values and their inference statistics.

### 4.1 Return Autocorrelation and Time-Varying Higher Distributional Moments

The website of Kenneth R. French[1] provides data for the empirical analysis performed in the present study. The market portfolio is approximated by the return series of the value-weighted market index $R_M$ from the Center for Research in Security Prices (CRSP) and the risk-free asset is approximated by the one-month Treasury bill rate $R_F$. The analysis starts in January 1964 and ends in December 2010, producing 564 monthly and 11,833 daily returns. The study uses nominal returns following Benartzi & Thaler (1995) and Zeisberger et al. (2007).

**Table 1:** *Empirical properties of monthly CRSP market portfolio returns*
This table reports, in the upper panel, empirical estimates of the mean (MEA), variance (VAR), skewness (SKE) and excess kurtosis (KUR) of $R_M$ from January 1964 to December 2010. The lower panel reports the Ljung-Box test statistic $Q_7$ for the first 7 lags of the autocorrelation function and the value of the first lag of the autocorrelation function $ACF_1$ with appropriate p-values in parentheses for each of these statistics. $Q_7$ and $ACF_1$ are computed for $R_M$ and for the time series of the empirical central higher moments $VAR_M^*$, $SKE_M^*$ and $KUR_M^*$ of the monthly return distributions of the CRSP market portfolio obtained by the ASBA.

| | **MEA** | **VAR** | **SKE** | **KUR** |
|---|---|---|---|---|
| $\boldsymbol{R_M}$ | 0.009 | 0.002 | -0.548 | 1.993 |
| | $\boldsymbol{R_M}$ | $\boldsymbol{VAR_M^*}$ | $\boldsymbol{SKE_M^*}$ | $\boldsymbol{KUR_M^*}$ |
| $\boldsymbol{Q_7}$ | 8.923 | 270.610 | 108.346 | 118.143 |
| | (0.258) | (0.000) | (0.000) | (0.000) |
| $\mathbf{ACF_1}$ | 0.087 | 0.467 | 0.377 | 0.387 |
| | (0.039) | (0.000) | (0.000) | (0.000) |

Table 1 reports empirical, unconditional estimates of the mean (MEA), variance (VAR), skewness (SKE) and excess kurtosis (KUR) of $R_M$. The table uses a monthly evaluation frequency because this investment horizon is the basis for the bootstrap approach of Zeisberger et al. (2007). Furthermore, Table 1 provides the Ljung-Box test statistic $Q_7$, which checks if the first 7 lags of the autocorrelation function equal zero as well as the value of the first lag of the autocorrelation function $ACF_1$. Appropriate p-values are placed in parentheses for each of these statistics. The statistics $Q_7$ and $ACF_1$ are computed for $R_M$ and for the time series of empirical estimates of the central variance $VAR_M^*$, central skewness $SKE_M^*$ and central kurtosis $KUR_M^*$. These distributional moments are computed based on the empirical one-month return distributions obtained in Step 6 of the ASBA, to which 1,000,000 bootstrap replications were applied.

The time series of the CRSP market portfolio returns has a positive MEA and a moderate VAR. Although not reported in Table 1, the monthly mean excess market return is 0.004, which implies an equity premium of 0.048 per annum. Furthermore, the unconditional return

[1] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

distribution of the CRSP market portfolio is skewed slightly left and leptokurtic in terms of the negative SKE of -0.548 and the positive KUR of 1.993. As a consequence, the time series of the monthly CRSP market portfolio returns shows pronounced higher distributional moments.

The test statistic $Q_K$ is computed as in Ljung & Box (1978). This test examines the null hypothesis of temporal independence of the respective time series of length $T$ for the first $K$ lags of the autocorrelation function where $K = \lceil \ln(T) \rceil$. The latter is motivated by Tsay (2005) and guarantees on the one hand that $Q_K$ is approximately $\chi_K^2$ distributed with $K$ degrees of freedom and on the other hand that the test has enough power to correctly reject the null hypothesis of temporal independence. Consequently, the Ljung-Box test does not allow any statements regarding the short-term persistence of $R_M$ because the examined time series of $R_M$ consists of 564 observations and thus $K = 7$. Therefore, Table 1 also reports $\mathrm{ACF}_1$ with appropriate p-values, which are based on the standard normally distributed test statistic $\mathrm{ACF}_1/\left(1/\sqrt{T}\right)$.

The first p-value of $Q_7$ in Table 1 shows that $R_M$ is not affected by autocorrelation, against which the values of $Q_7$ are very high for $VAR_M^*$, $SKE_M^*$ and $KUR_M^*$. The null hypothesis of temporal independence is rejected at a 99% significance level for these three distributional moments. Although not shown in Table 1, the time series of the standardized versions of the empirical third and fourth moments are also autocorrelated at a similar significance level as before. However, the respective values of $Q_7$ decrease to 85.644 and 38.801. Nevertheless, $Q_7$ thus far prohibits statements regarding short-term persistence. The latter is of particular interest because of the findings of Benartzi & Thaler (1995) on the substantiality of the autocorrelation of $R_M$.

In contrast to the Ljung-Box test, $\mathrm{ACF}_1$ in Table 1 indicates that $R_M$ is autocorrelated at lag 1 at a 95% significance level with a value of 0.087 and the respective p-value of 0.039. Against which, $\mathrm{ACF}_1$ confirms the results of the Ljung-Box test for the other time series. Therefore, $R_M$ seems to be affected by weak but significant short-term persistence when comparing the levels of $\mathrm{ACF}_1$ to each other.

As a conclusion, the monthly time series of $R_M$, $VAR_M^*$, $SKE_M^*$ and $KUR_M^*$ are significantly autocorrelated, in which $R_M$ shows only weak short-term persistence. The finding for $R_M$ is also supported by Benartzi & Thaler (1995). Furthermore, the findings of Bollerslev (1986), Harvey & Siddique (1999) and Brooks et al. (2005) agree with the above empirical evidence for the autocorrelation of $VAR_M^*$, $SKE_M^*$ and $KUR_M^*$. Consequently, application of the ASBA should produce different CPT value estimates and inference statistics for the excess return of the CRSP market portfolio than the approach of Zeisberger et al. (2007).

### 4.2 Comparing Approaches

The CPT is implemented empirically with (I) the ASBA and with (II) the approach of Zeisberger et al. (2007), which produce the respective CPT values $\hat{V}_n^I(R_M^e)$ and $\hat{V}_n^{II}(R_M^e)$. In Step 5, the ASBA uses 1,000,000 bootstrap replications to approximate the *n*-month return distributions. The approach of Zeisberger et al. (2007) is applied with 1,000,000 bootstrap

replications for the CPT value estimates computed using the method of Benartzi & Thaler (1995) and with 5,000×1,000,000 bootstrap replications for the associated inference statistics.

The threshold value in the weighting and value functions is a chained $n$-month return computed on the basis of the one-month Treasury bill rate. Therefore, the threshold values differ between the two approaches because for the ASBA, daily returns of the risk-free asset for each $n$-month window are chained, while in the approach of Zeisberger et al. (2007), monthly returns are chained according to the length of the investment horizon and subsequently averaged. Consequently, the threshold value varies across the $n$-month windows for the ASBA but stays constant over time when using the approach of Zeisberger et al. (2007). Nevertheless, the variation is not substantial and the methodical differences between the two types of threshold values do not affect the succeeding results.

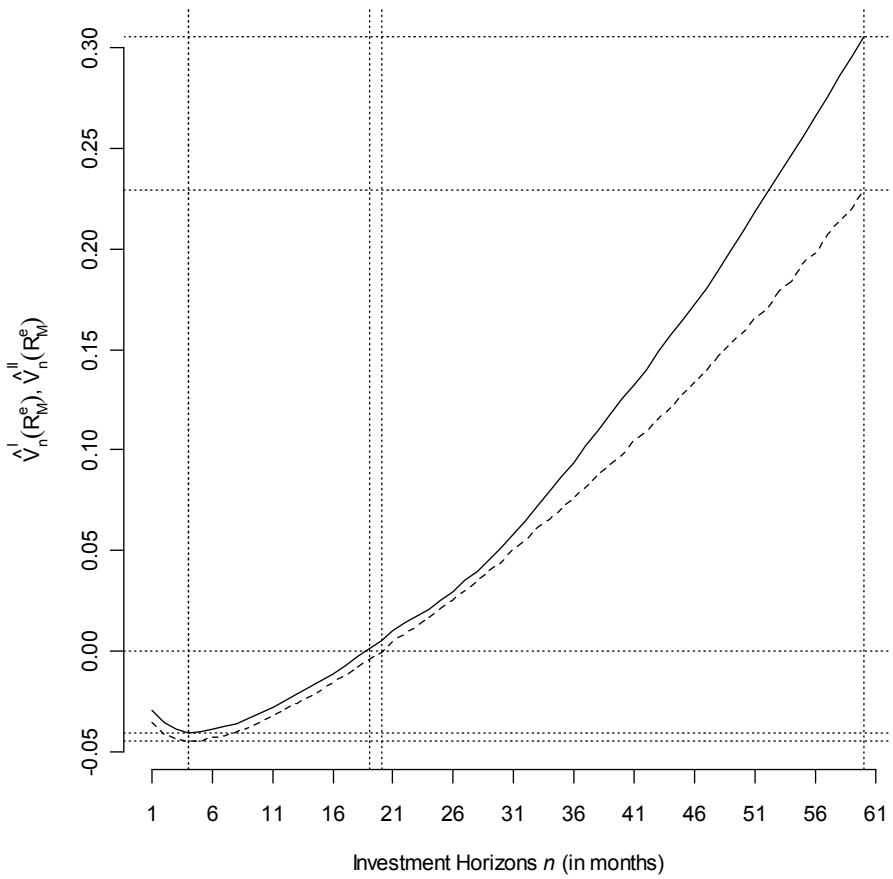


**Figure 1:** *Comparison of CPT values*
This figure illustrates $\hat{V}_n^I(R_M^e)$ as a solid line and $\hat{V}_n^{II}(R_M^e)$ as a dashed line for investment horizons $n = 1, \ldots, 60$ months. The underlying return series of $R_M^e$ starts in January 1964 and ends in December 2010. The vertical and horizontal dotted lines highlight various CPT values of interest.

Figure 1 compares $\hat{V}_n^I(R_M^e)$ as a solid line derived according to the ASBA, and $\hat{V}_n^{II}(R_M^e)$ as a dashed line derived according to the approach of Zeisberger et al. (2007) for investment horizons $n$ from 1 to 60 months. The underlying excess return series of the CRSP market portfolio $R_M^e$ starts in January 1964 and ends in December 2010. The estimated CPT values from the ASBA are computed in (9) with equal weights in which $w_i = 1$ for all $i =$

$1, \ldots, 564 - (n-1)$. The vertical and horizontal dotted lines highlight various CPT values of interest.

Both CPT value curves in Figure 1 have the same global minimum for an evaluation frequency of 4 months and the same global maximum at an investment horizon of 60 months, which also holds for the absolute global maximum. However, $\hat{V}_n^I(R_M^e)$ has its absolute global minimum at an investment horizon of 19 months against which $\left|\hat{V}_n^{II}(R_M^e)\right|$ is lowest at an investment horizon of 20 months. In addition, the curve of $\hat{V}_n^I(R_M^e)$ in Figure 1 lies above the curve of $\hat{V}_n^{II}(R_M^e)$ across all investment horizons and the two curves begin to drift apart starting with an investment horizon of 30 months. The former implies that the differences between $\hat{V}_n^I(R_M^e)$ and $\hat{V}_n^{II}(R_M^e)$ might not be driven by randomness because randomness would, for example, cause $\hat{V}_n^I(R_M^e) - \hat{V}_n^{II}(R_M^e)$ to switch signs across the investment horizons. Furthermore, the increasing differences of higher evaluation frequencies suggest that some time series characteristics become more pronounced under longer investment horizons. The next subsection provides evidence that the autocorrelation in the daily returns series is responsible for these increasing differences.

In contrast, Benartzi & Thaler (1995) and Zeisberger et al. (2007) found the absolute global minimum for an evaluation frequency of 11 months. Two things might explain this difference: The different estimation periods, particularly in terms of the finding of Zeisberger et al. (2007) that CPT values are sensitive towards the chosen time horizon of the underlying return series, and the fact that Benartzi & Thaler (1995) and Zeisberger et al. (2007) compute the differences in the CPT values of the market portfolio and the risk-free asset (against which this study evaluates the excess market return as motivated by the equilibrium model propounded by Barberis & Huang (2008)).

However, point-wise paired inference statistics would be necessary to check the null hypothesis $\hat{V}_n^I(R_M^e) = \hat{V}_n^{II}(R_M^e)$. These types of inference statistics cannot be obtained because on the one hand $\hat{V}_n^I(R_M^e)$ and $\hat{V}_n^{II}(R_M^e)$ are likely to be correlated because they rely on the same underlying return series, while on the other hand the two CPT value distributions differ in their dimensions (time-ordered versus unordered). Therefore, the differences between the two approaches are assessed via the evaluation of confidence intervals (CI) regarding the extent of the effect of myopic loss aversion.

**Table 2:** *Confidence intervals for CPT value estimates*
This table reports 90%, 95% and 99% CI, which do not contain 0, for the point estimates of the CPT values $\hat{V}_n^I(R_M^e)$ and $\hat{V}_n^{II}(R_M^e)$. All numbers are in percent.

| **n** | **1** | **2** | **3** | **4** | **5** | **6** | **7** |
|---|---|---|---|---|---|---|---|
| $\hat{V}_n^I(R_M^e)$ | -2.9 | -3.5 | -3.9 | -4.0 | | | |
| **90%** | [-3.6, -2.2] | [-5.0, -2.1] | [-6.4, -1.3] | [-7.7, -0.4] | | | |
| **95%** | [-3.8, -2.0] | [-5.3, -1.8] | [-6.8, -0.9] | | | | |
| **99%** | [-4.1, -1.8] | [-5.8, -1.2] | | | | | |
| $\hat{V}_n^{II}(R_M^e)$ | -3.5 | -4.1 | -4.3 | -4.5 | -4.4 | -4.3 | -4.2 |
| **90%** | [-4.4, -2.7] | [-5.6, -2.7] | [-6.2, -2.4] | [-6.9, -2.1] | [-7.4, -1.5] | [-7.6, -0.7] | [-8.0, -0.3] |
| **95%** | [-4.6, -2.5] | [-5.8, -2.5] | [-6.6, -2.0] | [-7.4, -1.6] | [-7.8, -1.0] | [-8.2, -0.0] | |
| **99%** | [-4.9, -2.1] | [-6.3, -2.0] | [-7.4, -1.3] | [-8.2, -0.6] | | | |

Table 2 reports 90%, 95% and 99% CI for $\hat{V}_n^I(R_M^e)$ and $\hat{V}_n^{II}(R_M^e)$. The CI for $\hat{V}_n^I(R_M^e)$ are computed with robust standard errors, as suggested by Andrews (1991), while the CI for $\hat{V}_n^{II}(R_M^e)$ are computed using the method of Trede (2002), as suggested by Zeisberger et al. (2007).

The results from Table 2 show that $\hat{V}_n^I(R_M^e) = 0$ cannot be rejected for evaluation frequencies of at least 5, 4 and 3 months at the respective significance levels of 90%, 95% and 99%, against which $\hat{V}_n^{II}(R_M^e) = 0$ holds for investment horizons of at least 8, 7 and 5 months. Therefore, both the ASBA and the approach of Zeisberger et al. (2007) produce myopic loss aversion. Furthermore, applying the ASBA implies another investor's perception of attractiveness because the CRSP market portfolio gains significantly in attractiveness against the one-month Treasury bill.

Benartzi & Thaler (1995) argue that an annual investment horizon is a natural interval over which fund managers and individual investors make and evaluate decisions. The length of this interval is predicated according to the publication frequency of major statement reports from companies and pension funds, among others. However, it is questionable to assume a single natural interval for all market participants because the amount of available information increases with the investment horizon. Therefore, one would expect that the market portfolio is either less attractive than the risk-free asset or equally attractive depending on whether the proportion of market participants who decide and evaluate decisions on base of this available information is significant, which in turn makes several natural intervals more likely.

As a consequence, the risky market portfolio should be less attractive than the risk-free asset for shorter investment horizons but equally attractive for more than a single investment horizon, such as the annual horizon discussed by Benartzi & Thaler (1995). Therefore, it seems more reasonable to assume that a substantial information exchange between market participants begins for example with publishing quarterly financial statement reports because the latter belong to the first regular major publications in a fiscal year. Thus, investors with at least a quarterly investment horizon should be indifferent between the risky market portfolio and the risk-free asset. The CI obtained by the ASBA imply an approximately quarterly switch from less attractiveness to equal attractiveness and thus, these CI support the assumed beginning of a substantial information exchange. In contrast, the CI from Zeisberger et al. (2007) imply that investors do not rely on quarterly information but only consider information on a semiannual or less frequent basis.

Furthermore, recalling that the null hypothesis $\hat{V}_n(R_M^e) = 0$ indicates a market in equilibrium for the respective investment horizon $n$ according to Barberis & Huang (2008), the effect of myopic loss aversion still explains the equity premium puzzle, particularly for one- and two-month evaluation frequencies because $\hat{V}_n(R_M^e) < 0$ holds here at a 99% significance level for both approaches. Hence, investors need a significant positive premium to compensate for the low attractiveness of the risky market portfolio against the risk-free asset.

### 4.3 Distinguishing Between the Effects

Although not shown in Table 2, applying, in Step 2, ($I^*$) a modified version of the ASBA with the autoregressive order $\hat{p}$ set to 0 only changes the results at a 90% significance level. Therefore, the CRSP market portfolio becomes less attractive than the risk-free asset for investment horizons of up to 3 months instead of 4 months as previously reported, although the null hypothesis of temporal independence of the daily return series is rejected at a 99% significance level with $Q_{10} = 77.730$. This study concludes that neglecting the temporal dependence in daily returns marginally affects the CI in Table 2, although the identified persistence of the daily returns of the CRSP market portfolio is more pronounced than the autocorrelation in the respective monthly return series. Consequently, neglecting the weaker short-term persistence of the monthly returns is not believed to be responsible for the differences shown in Figure 1 and Table 2, against which, allowing for time-varying higher distributional moments seems to substantially affect the CI in Table 2.

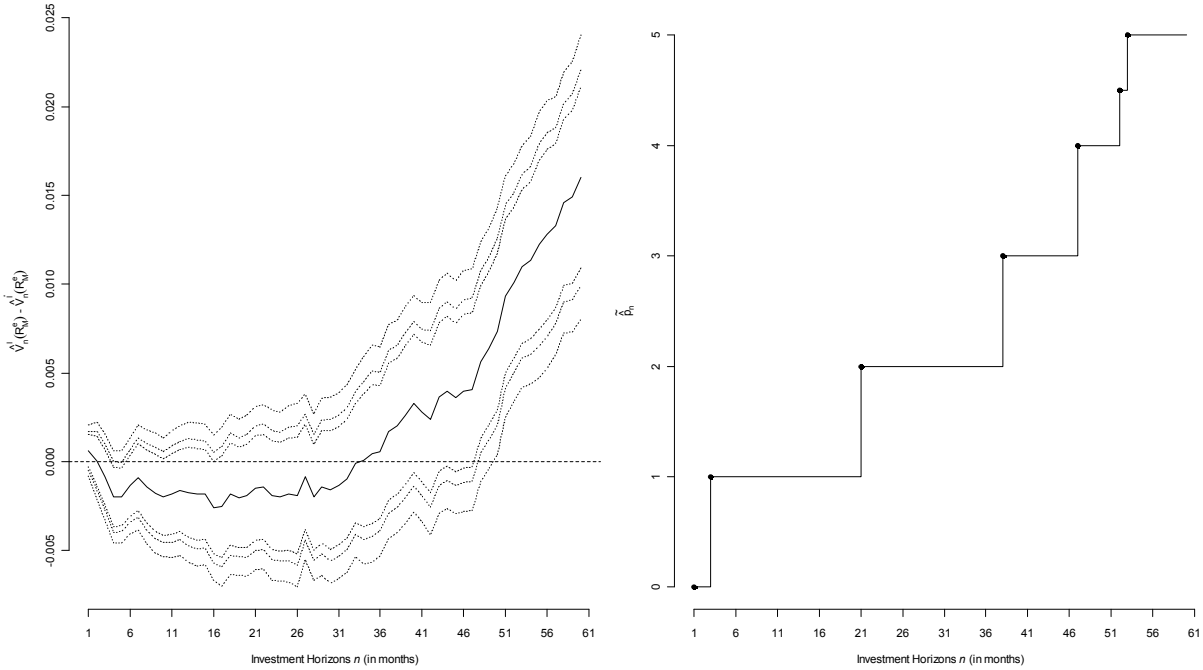


**Figure 2:** *The effects of autocorrelation on CPT value estimates*
This figure illustrates, in the left panel, $\hat{V}_n^I(R_M^e) - \hat{V}_n^{I^*}(R_M^e)$ as a solid line for investment horizons $n = 1, \ldots, 60$ months. The underlying return series of $R_M^e$ starts in January 1964 and ends in December 2010. The dotted lines denote the 90%, 95% and 99% CI for $\hat{V}_n^I(R_M^e) - \hat{V}_n^{I^*}(R_M^e)$ from inside to outside. The horizontal dashed line highlights the zero level. The right panel of this figure shows the median of the estimated autoregressive orders $\tilde{\hat{p}}_n$ of daily returns for investment horizons $n = 1, \ldots, 60$.

Figure 2 depicts in the left panel $\hat{V}_n^I(R_M^e) - \hat{V}_n^{I^*}(R_M^e)$ as a solid line and the corresponding CI as dotted lines, which are based on robust standard errors according to Andrews (1991), to decide whether the null hypothesis $\hat{V}_n^I(R_M^e) - \hat{V}_n^{I^*}(R_M^e) = 0$ holds. The CI are for the 90%, 95% and 99% significance levels, respectively, from inside to outside. The horizontal dashed line highlights the zero level. The right panel of Figure 2 illustrates the median autoregressive orders of daily returns $\tilde{\hat{p}}_n$ identified by the information criterion of Akaike (1974) for investment horizons $n = 1, \ldots, 60$.

The CI in Figure 2 reveal that $\hat{V}_n^I(R_M^e) - \hat{V}_n^{I^*}(R_M^e) = 0$ can be rejected for $n = 4, 5, 48, \ldots, 60$ at the 90% and 95% significance levels and for $n = 50, \ldots, 60$ at the 99% significance level. Apart from the two exceptions for the 90% and 95% CI for 4 and 5 months, considering autocorrelation in the daily returns do not seem to affect CPT value estimates significantly until an investment horizon of at least 48 months. Nevertheless, the increasing $\hat{V}_n^I(R_M^e) - \hat{V}_n^{I^*}(R_M^e)$ for an evaluation frequency of at least 30 months and the CI support the results from Figure 1 where $\hat{V}_n^I(R_M^e)$ and $\hat{V}_n^{II}(R_M^e)$ drift apart for investment horizons of 30 months or longer.

The latter finding becomes more evident in the right panel of Figure 2, which demonstrates that after an investment horizon of 30 months the step length becomes shorter and $\tilde{\hat{p}}_n$ grows faster. Therefore, there is evidence that the identified persistence in the daily returns affect the estimates of the CPT values of longer investment horizons because autocorrelation patterns in the return series become more pronounced. This is also consistent with the idea of the AR-sieve bootstrap because $p(T)$ in (8) increases with the time series length $T$, which in turn indicates more complex autocorrelation structures in the underlying return series.

Furthermore, $\hat{p}$ for the whole time series of 564 monthly return observations of the CRSP market portfolio is only 1 according the information criterion of Akaike (1974). Comparing this level of $\hat{p}$ with the levels of $\tilde{\hat{p}}_n$ in the right panel of Figure 2 further indicates that the autocorrelation in the monthly return series is not substantial, although it is significant as shown in Table 1. Therefore, further evidence demonstrates that considering time-varying higher distributional moments affects the CPT value estimates and their inference statistics because $\hat{V}_n^I(R_M^e) - \hat{V}_n^{II}(R_M^e)$ is positive across all investment horizons, as shown in Figure 1, while autocorrelation affects the results only when more pronounced, as shown in Figure 2.

In addition, the monthly time series of $\hat{V}_n^{i,I}(R_M^e)$ obtained by the ASBA provides further evidence of the influence of time-varying higher distributional moments on CPT value estimates because autocorrelation in the daily returns has not yet exerted a significant influence on the CPT value estimates, as shown in Figure 2, while $\hat{V}_n^{i,I}(R_M^e)$ shows significant autocorrelation patterns at a 95% significance level. The latter is indicated by the Ljung-Box statistic $Q_7 = 17.668$ with the appropriate p-value of 0.014. This significant temporal dependence supports the existence of the relationship between the ascertained time variation of higher distributional moments in Table 1 and the time variation of CPT values. This temporal dependence, in turn, promotes the differences between $\hat{V}_n^I(R_M^e)$ and the time-invariant estimator $\hat{V}_n^{II}(R_M^e)$.

## 5. Conclusion

The purpose of this study was to ascertain whether considering autocorrelation in the underlying return series and time-varying higher distributional moments affects the estimates and inference statistics of cumulative prospect theory values. Therefore, this study provided an approach to empirically implement the cumulative prospect theory while considering autocorrelation and allowing for time-varying higher distributional moments. An empirical

analysis compared the proposed approach with the procedure of Zeisberger et al. (2007) by reevaluating the effect of myopic loss aversion.

The results of this study showed that using the autoregressive sieve bootstrap algorithm instead of the approach of Zeisberger et al. (2007) produces a gain in attractiveness of the market portfolio against the risk-free asset. The results also showed that this additional gain is mostly attributed to the neglect of time-varying higher moments of the return distribution of the market portfolio. In contrast, considering return autocorrelation affects the level of the cumulative prospect theory values only for longer investment horizons because serial correlation patterns in the return series become more pronounced. Nevertheless, both approaches support the effect of myopic loss aversion and thus cumulative prospect theory investors find the market portfolio less attractive than the risk-free asset at a 95% significance level for investment horizons of 1 to 3 months for the proposed approach and for evaluation frequencies of 1 to 6 months for the procedure of Zeisberger et al. (2007).

In conclusion, when applying the cumulative prospect theory to evaluate investment opportunities, which are based on time series data, time-varying higher distributional moments and the autocorrelation of returns, particularly when more pronounced, render it valuable to apply the autoregressive sieve bootstrap algorithm presented in this study because both time series characteristics affect the estimated perception of attractiveness of cumulative prospect theory investors.

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