Forecasting Dynamic Hedge Ratios and Value at Risk Using GARCH Models: Evidence From SP 500 FTSE 100 and NIKKEI 225

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ABSTRACT

This paper investigates the behaviour of dynamic hedge ratios in three markets, using five principal variants of GARCH models and compares the forecasting performance of optimal hedge ratios across those models. More specifically, using daily data of the spot and futures contracts of SP 500, FTSE 100, and NIKKEI 225 within the framework of standard GARCH, GARCH-BEKK, GARCH-ECM, GARCH-X, and asymmetric GARCH models, we have estimated the time-varying hedge ratios and compared the forecasting performances of those hedge ratios. We have also computed the minimum capital risk requirement (MCRR) using those hedge ratios to ascertain the superiority of alternative hedging strategy that holds capital adequacy requirement of the fund at a minimum level. Results based on SP 500 data show that MCRR estimates obtained from the GARCH-BEKK performs best among the GARCH class of models in most cases both for short and long hedge. In the case of FTSE 100, GARCH-BEKK uniformly performs best for short hedge, whilst standard GARCH performs better for long hedge in most cases. In case of NIKKEI 225, GARCH-X outperforms other competing models for short hedge in most cases, whilst the standard GARCH performs better in most cases for long hedge. Our comparative results suggest that position in NIKKEI 225 contract is slightly more risky than SP 500 and FTSE 100 contracts.

Key Words: Stock index futures; time varying hedge ratio; GARCH models; forecasting, minimum capital risk requirement..

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1. Introduction

Since the seminal work of Working (1953), Johonson (1960), and Ederington (1979), there has been tremendous interest over the last half century towards the modelling and forecasting of the optimal hedge ratios (OHR) and alternative hedging strategies applied to the commodity and financial futures. It is now well-known that the principal functions of futures markets are price discovery, hedging, speculation and risk-sharing. Hedgers use these markets as a means to avoid the risk associated with adverse price change in the related cash markets. A hedge is performed by taking simultaneous positions both in cash and future markets-which result in the offset of any loss sustained from an adverse price movement in one market, by a favourable price movement in another market. The hedge ratio is simply the number of futures contracts needed to minimize the exposure of a unit worth position in the cash market. Therefore, an investor holding a long position in the cash market should short h units of futures contracts, where h would be the hedge ratio.

Earlier studies during 1960s and early 1970s, employed the OLS based regression analysis assuming that the optimal hedge ratio is time invariant [see Johnson (1960) and Ederington (1979)]. However, it is now well established that most asset return distributions are not normal, i.e., return distributions are time varying with high skewness and high excess kurtosis. As a consequence, the hedge ratio is also changing over time. The development of GARCH modelling technique, to deal with time varying volatility, has generated renewed interest in the empirical investigation of the effectiveness of dynamic hedging that allows the hedge ratio to be time

varying.[1] Consequently, a large body of empirical literature has accumulated since late 1980s and up to recent years examining the issues of relative effectiveness of sophisticated hedging method over much simpler and intuitively appealing traditional hedging methods-using currencies, commodities, stock indices, and interest rate products using ARCH and GARCH specifications. Based on the evidence of time-varying distributions of spot and future prices, the dynamic hedging strategy has proved superior to alternative hedging strategy that holds the hedge ratio constant.

Given the plethora of literature, there are serious gaps in the current research strand in two directions. Firstly, previous research has evaluated the relative effectiveness of alternative hedging strategies by examining the in-sample and out-of-sample performance of variance reduction of portfolios of returns in the cash and futures markets based on the information of hedge ratios, spot and futures prices of the assets. Surprisingly, however, from a risk management perspective, there has been no attempt to evaluate the forecasting accuracy and performance of the estimated hedge ratios derived from those econometric models. Indeed, knowledge of forecasting ability of optimal hedge ratio/dynamic hedge ratio is important for understanding the role of futures markets in equity trading, program trading, index arbitrage and the development of optimal hedging and trading strategies in fund management. Careful selection of derivatives contracts is conditional upon the accuracy of OHR estimates and volatility forecasting techniques. Secondly, given that hedge ratios of various portfolios are predictable, an investor always prefers a portfolio with a lower financial capital to reach the maximum of risk reduction. The determination of optimal hedge ratio helps the investor to choose the optimal portfolios with suitable futures and a reasonable number of futures contracts. There is

[1]See Choudhry (2009) and Kroner and Sultan (1993) and a list of references therein.

scanty research which conducts a comparative evaluation of the minimum capital risk requirements (MCRR) using those hedge ratios that holds capital adequacy requirement of the fund at a minimum level..

This article, therefore, investigates the behaviour of dynamic hedge ratios in three markets, using five principal variants of GARCH models and compares the forecasting performance of optimal hedge ratios across those models. More specifically, using daily data of the spot and futures markets of USA, UK, and Japan within the framework of standard GARCH, GARCH-BEKK, GARCH-ECM, GARCH-X, and asymmetric GARCH models, we have estimated the time-varying hedge ratios and compared the forecasting performances of those hedge ratios. We have also computed the minimum capital risk requirement (MCRR) using those hedge ratios to ascertain the superiority of alternative hedging strategy that holds capital adequacy requirement of the fund at a minimum level.

We have chosen the stock index futures contracts of those countries in our investigation. Stock index futures contracts, in particular, offer opportunities to unbundling the market and non-market components of risk and return to investment banks, security houses, fund managers and individual investors. For example, stock index futures are routinely used in program trading and index arbitrage to achieve portfolio insurance or arbitrage purpose by exploiting the relationship between the cash value of the index and the futures on the index. Fund managers use them to alter, temporarily, the systematic risk of a portfolio without having to buy or sell its constituent stock. The paper is organized as follows. Section 2 describes and discusses the optimal hedge ratio and the five GARCH models. The data and preliminary diagnostics are described in Section 3. Sections 4-7 offer the empirical

results based upon estimating conventional and dynamic hedging models and the final section offers a summary and conclusion.

2. Estimation of Optimal Hedge Ratios and the GARCH Models

2.1. The Hedge Ratio

Johnson's (1960) risk minimizing hedge ratio h^* is defined as:

$$h^* = -\frac{\sigma_{c,f}}{\sigma_f^2} = -\frac{\mathrm{cov}(R_c, R_f)}{\mathrm{var}(R_f)} \tag{1}$$

where R_c and R_f denotes return on spot and future indices. The optimal hedge ratio (OHR) then is computed as the slope coefficient of the following regression:

$$R_{ct} = \alpha + \beta R_{ft} + \varepsilon_t \tag{2}$$

where ε_t is an error term.[2] A $\beta = 0$ implies unhedged position; $\beta = 1$ signifies a fully hedged position; and $\beta < 1$ implies a partial hedge.

It is now well-known in the literature that the conventional hedging model has shortcomings. As the distribution of futures and spot prices are changing through time, h^* which is expressed as the ratio of covariance between futures returns and cash returns and variance of futures returns, moves randomly through time [Checchetti et al. (1988), Baillie and Myers (1991), Kroner and Sultan (1993)]. Therefore eq.(2) should be modified as:

$$h_T^* = \frac{\mathrm{cov}(R_{T+1}^c, R_{T+1t}^f \mathrm{I}\Omega_T)}{\mathrm{var}(R_{T+1}^f \mathrm{I}\Omega_T)} \tag{3}$$

[2] The OLS estimation of the hedge ratio from equation (2) is based on the assumption of time invariant asset distributions suggested by Ederington (1979), and Anderson and Danthine (1980).

In eq. (3), conditional moments are changing as the information set, Ω_T, is updated, consequently, the number of futures contracts held and the optimal hedge ratio will also change through time-hence the t subscripts of h_T^*. Under the condition of time-varying distribution, the bivariate GARCH model is utilized to estimate the time-varying hedge ratios to approximate the dynamic hedging strategies.

2.2. Bivariate GARCH Model

The time-varying hedge ratios are estimated from five variants of bivariate GARCH models: standard GARCH, GARCH-ECM, GARCH-BEKK, GARCH-GJR and GARCH-X. The following bivariate GARCH(p,q) model is applied to returns from the stock cash and futures markets:

$$y_t = \mu + \varepsilon_t \tag{4}$$

$$\varepsilon_t / \Omega_{t-1} \sim N(0, H_t) \tag{5}$$

$$vech(H_t) = C + \sum_{i=1}^{p} A_i vech(\varepsilon_{t-i})^2 + \sum_{j=1}^{q} B_j vech(H_{t-j}) \tag{6}$$

Where $y_t = (r_t^c, r_t^f)$ is a (2x1) vector containing stock returns from the cash and futures markets. H_t is a (2x2) conditional covariance matrix, C is (3x1) parameter vector of constant, A_i and B_j are (3x3) parameter matrices, and vech is the column stacking operator that stacks the lower triangular portion of a symmetric matrix

To make the estimation amenable, Engle and Kroner (1995) have suggested various restrictions to be imposed on the parameters of A_i and B_j matrices. A parsimonious representation may be achieved by imposing a diagonal restriction on the parameter matrices so that each variance and covariance element depends only on its own past values and prediction errors. The following equations represent a diagonal vech bivariate GARCH(1,1) conditional variance equation(s):

$$H_{11,t} = C_1 + A_{11}(\varepsilon_{1,t-1})^2 + B_{11}(H_{11,t-1}) \tag{7a}$$

$$H_{12,t} = C_2 + A_{22}(\varepsilon_{1,t-1}, \varepsilon_{2,t-1}) + B_{22}(H_{12,t-1}) \tag{7b}$$

$$H_{22} = C_3 + A_{33}(\varepsilon_{2,t-1})^2 + B_{33}(H_{22,t-1}) \tag{7c}$$

In the bivariate GARCH(1,1) model, the diagonal vech parameterization involves nine conditional variance parameters.

Using the bivariate GARCH model, the time-varying hedge ratio can be computed as:

$$h_t^* = \hat{H}_{12,t} / \hat{H}_{22,t} \tag{8}$$

Where $\hat{H}_{12,t}$ is the estimated conditional covariance between the cash and futures returns, and $\hat{H}_{22,t}$ is the estimated conditional variance of futures returns. Since, the conditional covariance is time-varying the optimal hedge would be time-varying too.

2.3. GARCH-ECM Model

When the bivariate GARCH model incorporates the error correction term in the mean equation, it becomes the GARCH-ECM model which is presented as:

$$y_t = \mu + \delta(u_{t-1}) + \varepsilon_t \tag{9}$$

Where u_{t-1} denotes the lagged error-correction term, retrieved from the cointegration regression. Therefore, a bivariate GARCH-ECM model will be employed to account for the long-run relationship and basis risk (see, Kroner and Sultan 1993). Equation 8 still represents the hedge ratio.

2.4. Bivariate GARCH-BEKK Model

In the BEKK model as suggested by Engle and Kroner (1995), the conditional covariance matrix is parameterized to:

$$vech(H_t) = C'C + \sum_{k=1}^{k}\sum_{i=1}^{p} A'_{ki}\varepsilon_{t-i}\varepsilon'_{t-1}A_{ki} + \sum_{k=1}^{k}\sum_{j=1}^{q} B'_{kj}H_{t-j}B_{kj} \tag{10}$$

Eqs. (8) and (9) also apply to the BEKK model and are defined as before. In eq(14) A_{ki}, $i = 1,...q$, $k = 1,...k$, and B_{kj} $j = 1,...q$, $k = 1,..k$ are NxN matrices. The GARCH-BEKK model is sufficiently general that it guarantees the conditional covariance matrix, H_t to be positive definite, and renders significant parameter reduction in the estimation. For example, a bivariate BEKK GARCH(1,1) pamerization requires to estimate only 11 parameters in the conditional variance-covariance structure. The time-varying hedge ratio from the BEKK model is again represented by equation 8.

2.5 Bivariate GARCH-GJR Model:

Along with the leptokurtic distribution of stock returns data, empirical research has shown a negative correlation between current returns and future volatility (Black, 1976; Christie, 1982). This negative effect of current returns on future variance is sometimes called the leverage effect (Bollerslev et al. 1992). Glosten et al. (1993) provide a modification to the GARCH model that allows positive and negative innovations to returns to have different impact on conditional variance.[3] Glosten et al. (1993) suggest that the asymmetry effect can also be captured simply by incorporating a dummy variable in the original GARCH.

$$\sigma_t^2 = \alpha_0 + \alpha u_{t-1}^2 + \gamma u_{t-1}^2 I_{t-1} + \beta \sigma_{t-1}^2 \qquad (11)$$

Where $I_{t-1} = 1$ if $u_{t-1} > 0$; otherwise $I_{t-1} = 0$. Thus, the ARCH coefficient in a GARCH-GJR model switches between $\alpha + \gamma$ and α, depending on whether the lagged error term is positive or negative. The time-varying hedge ratio based on the GARCH-GJR model is also expressed as equation 8.

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[3] There is more than one GARCH model available that is able to capture the asymmetric effect in volatility. According to Engle and Ng (1993), the Glosten et al. (1993) model is the best at parsimoniously capturing this asymmetric effect.

2.6 Bivariate GARCH-X Model:

The GARCH-X model is an extension of the GARCH-ECM model as it incorporates the square of error correction term in the conditional covariance matrix. In the GARCH-X model, conditional heteroscedasticity may be modelled as a function of lagged squared error correction term-in addition to the ARMA terms in the variance/covariance equations:

$$vech(H_t) = C + \sum_{i=1}^{p} A_i vech(\varepsilon_{t-i})^2 + \sum_{j=1}^{q} B_j vech(H_{t-j}) + \sum_{k=1}^{k} D_k vech(u_{t-1})^2 \quad (12)$$

A significant positive effect may imply that the further the series deviates from each other in the short run, the harder they are to predict. The hedge ratio again is presented by equation 8.

It is hypothesized that time-varying hedge ratios would be different across different variants of GARCH models. Therefore, the next question arises: which one is more effective? As stated earlier in this paper we apply all the above methods to estimate the hedge ratio, and compare their effectiveness. We also compare the hedging performance of dynamic hedging strategies with traditional hedging methods.

3. Data and Diagnostics

The model is evaluated using daily data on stock indices and their counterpart futures contracts from SP500, FTSE100 and NIKKEI225. Empirical evaluation of hedging performance using daily data has tremendous value for money managers, who adjust their portfolio as often as daily (Figlewski 1986). Therefore, we have analysed the empirical models using daily data spanning the period July, 2000 to June, 2011. Following Park and Switzer (1995), and Choudhry (2003), of the several future outstanding contracts at any given time, we use the price of the nearest contract. The data are collected from Datastream.

Descriptive statistics relating to the distribution of return indices are presented in Table 1. These statistics are: mean, standard deviation, variance, a measure of skewness, a measure of excess kurtosis (normal value is 3), the Jarque-Bera statistics, and unit root test results of cash and future price indices. The table also presents higher order autocorrelation Q, and ARCH effects in the returns indices series. The values of the skewness statistics indicate that the density function is negatively skewed for both cash and future return indices for all markets except the future market of U.S.A. The values of the excess kurtosis statistic are more than 2 for all countries, which suggest that the density function for each country has a fat tail. The values of the Jarque-Bera statistic are high, suggesting the return indices are not normally-distributed. Judged by the skewness, excess kurtosis and Jarque-Bera statistics, it can be inferred that the return indices exhibit 'fat-tails' in all markets. The data series have also been checked for stationarity using the Elliott-Rothenberg-Stock Dickey-Fuller generalized least squares (DF-GLS) unit root test.[4] The DF-GLS test results indicate that each of the return indices series has no unit root. Tests for autocorrelation in the first moments using the $Q(20)$ statistic indicate that none is present in the any of the indices. Finally, tests for ARCH using Engle's (1982) LM statistic generally support the hypothesis of time-varying variances.

4. Empirical Results

In this section we evaluate formally the effectiveness of conventional and time varying hedging models described in the preceding section. First, Table 2 presents the OLS regression results of cash returns and future returns (equation 2) for all three

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[4]It is well known that Augmented Dickey-Fuller (ADF) and Phillips-Perron unit root tests have low power in rejecting the null of a unit root and are prone to size distortion. Elliott, Rothenberg and Stock (1996) proposed an alternative DF-GLS test which involves the application of a generalized least squares method to de-trend the data. In the process of performing this test, the autoregressive truncation lag length is determined by the modified Akaike Information Criterion (AIC).

markets obtained by using the Cochrane-Orcutt method. Here, daily spot changes in the index are regressed on daily changes in the nearby index futures contract. Parameter estimates of the future returns in eq. (2) represent the constant minimum variance hedge ratio (*t*-stats in parenthesis). In all cases, the coefficient attached to the future returns variable is positive and highly significant. The hedge ratio is found to be less than unity in all cases. The largest coefficient is found in the case of FTSE100 (.9865) and lowest for NIKKEI225 (.9502). This statistic indicates that a substantial portion of variability in the cash market is hedged using the futures instruments.

The static bivariate regression method of the OHR suffers from several potential limitations, in particular the omission of the basis (error correction term) as a determinant of the hedge ratio and accounting for time dependence in risk. Therefore, in the next step, we have estimated alternative variants of GARCH models to accommodate time dependence in variance/covariance and error correction term. The standard GARCH, GARCH-BEKK, GARCH-GJR, are estimated without the error correction term in the mean equation. GARCH-ECM incorporates the error correction term in the mean equation whereas the GARCH-X model incorporates the error correction term both in mean and variance equations.

We are currently conducting empirical estimation. Results with analysis would be available definitely before the conference date.

6. Measures of Forecast Accuracy

A group of measures derived from the forecast error are designed to evaluate *ex post* forecasts. To evaluate forecasts, different measures of forecast errors (MAE and MSE) are employed. Mean errors (ME) are employed to assess whether the models

over or under-forecast return series.[5] Among them, the most common overall accuracy measure is MSE (Diebold 2004, p. 298):

$$MSE = \frac{1}{n}\sum_{t=1}^{n} e_t^2 \tag{13}$$

$$MAE = \frac{1}{n}\sum_{t=1}^{n} |e_t| \tag{14}$$

$$ME = \frac{1}{n}\sum_{t=1}^{n} e \tag{15}$$

Where e is the forecast error defined as the difference between the actual value and the forecasted value.[6]

Diebold and Mariano (1995) develop a test of equal forecast accuracy to test whether two sets of forecast errors, say e_{1t} and e_{2t}, have equal mean value. Using MSE as the measure, the null hypothesis of equal forecast accuracy can be represented as $E[d_t] = 0$, where $d_t = e_{1t}^2 - e_{2t}^2$. Supposing, n, h-step-ahead forecasts have been generated, Diebold and Mariano (1995) suggest the mean of the difference between MSEs $\bar{d} = \frac{1}{n}\sum_{t=1}^{n} d_t$ has an approximate asymptotic variance of

$$Var(\bar{d}) \approx \frac{1}{n}\left[\gamma_0 + 2\sum_{k=1}^{h-1}\gamma_k\right] \tag{16}$$

Where γ_k is the k-th autocovariance of d_t, which can be estimated as:

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[5] A negative ME indicates a model under forecast and a positive ME indicates over forecasting.

[6] The lower the forecast error measure, the better the forecasting performance. However, it does not necessarily mean that a lower MSE automatically indicates superior forecasting ability, since the difference between the MSEs may be not significantly different from zero.

$$\hat{\gamma}_k = \frac{1}{n}\sum_{t=k+1}^{n}(d_t - \bar{d})(d_{t-k} - \bar{d}) \tag{17}$$

Therefore, the corresponding statistic for testing the equal forecast accuracy hypothesis is $S = \bar{d} / \sqrt{Var(\bar{d})}$, which has an asymptotic standard normal distribution. According to Diebold and Mariano (1995), results of Monte Carlo simulation experiments show that the performance of this statistic is good even for small samples and when forecast errors are non-normally distributed. However, this test is found to be over-sized for small numbers of forecast observations and forecasts of two-steps ahead or greater.

Harvey *et al.* (1997) further develop the test for equal forecast accuracy by modifying Diebold and Mariano's (1995) approach. Since the estimator used by Diebold and Mariano (1995) is consistent but biased, Harvey *et al.* (1997) improve the finite sample performance of the Diebold and Mariano (1995) test by using an approximately unbiased estimator of the variance of $\bar{d}$. The modified test statistic is given by

$$S^* = \left[\frac{n + 1 - 2h + n^{-1}h(h-1)}{n}\right]^{1/2} S \tag{18}$$

Through Monte Carlo simulation experiments, this modified statistic is found to perform much better than the original Diebold-Mariano test at all forecast horizons and when the forecast errors are autocorrelated or have non-normal distribution. In this paper, we apply the modified Diebold-Mariano test.

We are currently conducting empirical estimation. Results with analysis would be available definitely before the conference date.

7 Hedging Effectiveness Minimum Capital Risk Requirement:

Given that hedge ratios of various portfolios obtained from GARCH models are predictable, fund managers always prefer a portfolio with a lower financial capital. Over the past twenty years, financial institutions have been using economic models to identify and measure various markets risks using a variety of procedures and set aside a provision of capital for potential losses. One popular approach is the calculation of Value at Risk (VaR) using in-house economic model to estimate the Minimum Capital Risk Requirements (MCRR). In this section, we are evaluating hedging effectiveness by estimating and comparing Minimum Capital Risk Requirement (MCRR) for portfolio returns obtained under alternative hedging models.

The VaR is a statistical measure of expected maximum loss on the value of the trading position of the asset portfolio taken by an institution which is exposed to market risk. For expository convenience, we denote the value of the portfolio by R; the VaR can be derived from the probability distribution of the future portfolio as worst possible realization signified as R^* such that the probability of a value lower than R^* is:

$$P(R \leq R^*) = \int_{-\infty}^{R^*} f(R)dR = 1 - c \qquad (19)$$

where (1-c) represents the probability of lower-tail event, i. e., 5% for over 95% confidence level.[7] Following Brooks et al. (2002), if we define R^* in relation to a benchmark portfolio position such that:

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[7] Investors wish to find this worst possible realization R^* at a given significance level c, such that the probability of exceeding this value is:

$$c = \int_{R^*}^{\infty} f(R)dR \qquad (19a)$$

$$R^* = R_1 - R_0 \tag{20}$$

Where R_0 signifies the initial position; R_1 denotes lowest simulated value of the portfolio in case of a long futures position or the highest simulated value of the portfolio in the case of a short hedge position. The maximum expected loss as a proportion of initial value of the portfolio will depend on the probability distribution of R^*/R_0. Based on the assumption of lognormal distribution of asset price, the next step to find the fifth quantile of $In(R_1/R_0)$:

$$\frac{In(R_1/R_0) - \mu}{\sigma} = \pm\alpha \tag{21}$$

Where α is the fifth quantile from a standard normal distribution, μ is the mean of $In(R_1/R_0)$ and σ is the standard deviation of $\text{In}(R_1/R_0)$. Cross-multiplying, taking the exponential, and after certain manipulation:

$$R^* = R_0 \pm [1 - Exponential(\pm\alpha\sigma) + \mu] \tag{22}$$

We calculate the MCRR for 1-day, 10-day, 20-day, 30-day, 60-day, 90-day and 180-day investment horizons, by simulating densities of portfolio returns using Efron's (1982) bootstrapping methodology which is based on a multivariate GARCH(1,1) model.[8] The Monte Carlo simulation procedure used 10,000 simulated paths of portfolio returns based on a GARCH(1,1) model to generate an empirical distribution of the maximum loss.

The probability density function represented by the area under the curve ranges between $-\infty$ to ∞ sums up to one.

[8] Interested readers are referred to Brooks et. al. (2002), and Jorion (2007) for a detailed discussion.

Tables' 3-5 presents the estimates of the MCRR using hedge ratios obtained from the alternative hedging models. The top panel of Tables 3-5 presents MCRR for a short hedge (long cash and short futures) and the lower panel of the Tables 3-5 shows the results for a long hedge (long futures, short cash). The results are reported in units of index points. Table 3 presents the MCRR estimates based on the data of SP 500 indices. The results show that for short hedge, GARCH-BEKK performs best among the GARCH class of models in most cases. GARCH-GJR outperforms other competing models only at a long investment horizon (180 day). For long hedge, GARCH-BEKK outperforms other variants of GARCH models in most cases, while GARCH-ECM and standard GARCH outperform their competing models at 30-day and 90-day forecast horizons, respectively. The long hedge positions appear to require slightly more MCRR than comparable short hedge positions over the forecast horizons. Table 4 presents the MCRR estimates for the FTSE 100. In the case of FTSE 100, GARCH-BEKK uniformly performs best among the GARCH class of models for short hedge. For long hedge, standard GARCH performs better among the GARCH class of models in most cases, while the GARCH-ECM outperforms their competing models at 1-day and 10-day forecast horizons. GARCH-BEKK outperforms only at 180-day investment horizon in the case of long hedge. Table 5 presents the MCRR estimates of Nikkei 225. For short hedge, GARCH-X outperforms their competing models in four cases (10-day, 60-day, 90-day and 180-day forecast horizons); naïve hedge outperforms in two cases (20-day and 30-day forecast horizons). GARCH-BEKK outperforms best only at 1-day investment horizon. For long hedge, the standard GARCH outperforms other competing models in four cases (1-day, 30-day, 60-day and 90-day forecast horizons). GARCH-BEKK outperforms their competing models at 10-day and 180-day forecast horizons.

GARCH-GJR performs well at the 20-day investment horizon. When we compare results across Tables 3-5, it is evident that compared to positions in SP 500 and FTSE 100, position in NIKKEI 225 requires more capital both under short hedge and long hedge conditions.[9] These comparative results suggest that position in NIKKEI 225 contract is slightly more risky than SP 500 and FTSE 100 positions.

References

Anderson, R. W., and Danthine, J. P. (1980) Hedging and joint production: theory and illustrations, *Journal of Finance*, **35**, 489-497.

Baillie, R., and Myers, R. (1991), Bivariate GARCH estimates of the optimal commodity futures hedge, *Journal of Applied Econometrics*, **6**, 109-124.

Black, F. (1976). Studies of Stock Market Volatility Changes, *Proceedings of the American Statistical Association, Business and Economics Statistics Section*, pp. 177-181.

Bollerslev, T., Chou, R. and Kroner, K. (1992). ARCH Modeling in Finance, *Journal of Econometrics*, **52**, 5-59.

Brooks, C., Henry, O. T., and Persand, G. (2002) The effects of asymmetries in optimal hedge ratios, *Journal of Business*, **75**, 333-352.

[9] For example, at 1-day, 10-day, 90-day and 180-day forecast horizons, a short hedge in SP 500 (FTSE 100) position requires 1.61% (1.60%), 4.91% (4.90%), 16.49% (17.30%) and 25.12% (29.29%) of portfolio returns as capital, whereas similar positions in NIKKEI 225 require 1.98%, 6.39%, 24.85% and 41.33% portfolio returns as capital. A long hedge in SP 500 (FTSE 100) requires 1.91% (1.49%), 5.75% (4.59%), 18.56% (17.45%) and 26.57% (26.37%) of portfolio returns as capital at 1-day, 10-day, 90-day and 180-day forecast horizons. In comparison, positions in NIKKEI 225 require 1.92%, 6.23%, 24.76% and 39.44% of portfolio returns.

Cehetti, S. G., Cumby, R. E. and Figlewski, S. (1988) Estimation of optimal futures hedge, *Review of Economics and Statistics*, **70**, 623-630.

Choudhry, T. (2003), Short-run deviations and optimal hedge ratio: evidence from stock futures, *Journal of Multinational Financial Management*, **13**, 171-192.

Choudhry, T. (2009) Short-run deviations and time-varying hedge ratios: evidence from agricultural futures markets, *International Review of Financial Analysis*, **18**, 58-65.

Christie, A. (1982). The Stochastic Behavior of Common Stock Variances: Value, Leverage and Interest Rate Effects, *Journal of Financial Economics*, **10**, 407-432.

Diebold, F. X. (2004) *Elements of Forecasting*, Third Edition, Ohio: Thomson South-Western.

Diebold, F. X. and Mariano, R. S. (1995). Comparing Predictive Accuracy, *Journal of Business and Economic Statistics*, Vol., pp. 253-263.

Ederington, L. (1979) The hedging performance of the new futures markets, *Journal of Finance*, **34**, 157-170.

Engle, R. and Ng, V. (1993). Measuring and testing the impact of news on volatility, *Journal of Finance*, 48, pp. 1749-1778.

Engle, R. F. and Kroner, K. F. (1995). Multivariate Simultaneous GARCH, *Econometric Theory*, **11**, 122-150.

Figlewski, S. (1986) *Hedging with Financial Futures for Institutional Investors*, Ballinger Publishing, Cambridge, MA.

Glosten, L. R., Jagannathan, R., and Runkle, D. E. (1993) On the relation between the expected value and the volatility of the nominal excess return on stocks, *Journal of Finance*, 48, 1779-1801.

Harvey, D., Leybourne, S. J. and Newbold, P. (1997). Testing the equality of prediction mean squared errors, *International Journal of Forecasting*, Vol. 13, pp. 281–291.

Johnson, L. (1960), The theory of hedging and speculation in commodity futures, *Review of Economic Studies*, **27**, 139-151.

Jorion, P., 2007. Value at Risk: The New Benchmark for Managing Financial Risk. Third edition, McGraw-Hill, New York.

Kroner, K. F. and Sultan, J. (1993), Time-varying distributions and dynamic hedging with foreign currency futures, *Journal of Financial and Quantitative Analysis*, **28**, 535-51.

Working, H. (1953) Futures trading and hedging, *American Economic Review*, **43**, 314-343.

Table 1: Descriptive statistics of stock spot and futures indices return

Statistics	SP500		FTSE100		NIKKEI225	
	Cash Return	Future Return	Cash Return	Future Return	Cash Return	Future Return
Mean	-.000045	-.000052.	-.000037	-.000041	-.000213	-.000211
Variance	.000176	.000179	,000166	.000165	.000248	.000248
Std. Dev.	.013274	.013367	,012889	.012834	.015763	.016356
Skewness	-.115422	.049888	-.131169	-.154715	-.386679	-212760
Kurtosis	8.55974	11.0187	6.744135	7.244107	7.268245	13.0235
Jarque-Bera	8691.77	14393.74	5399.83	6232.07	6333.14	20127.52
Stationarity: t_μ	-14.43	-13.88	-9.32	-8.86	-6.43	-4.45
t_τ	-18.06	-17.66	-15.14	-13.77	-10.62	-8.09
ARCH(1)	91.21	76.33	162.86	150.45	87.33	26.74
Q(20)	75.03	64.42	106.33	106.33	21.10	40.01
Corr($\Delta s_t, \Delta f_t$)		.9796		.9846		.9700

Note: t_μ and t_τ are the Elliot-Rothenberg-Stock Dickey-Fuller generalised least squares (DF-GLS) unit root test statistics with allowance for a constant and trend, respectively. 5% critical values of t_μ and t_τ are -1.940 and -2.890 (see Elliot-Rothenberg-Stock 1996, Table 1).

Table 2: Bivariate regression results of the constant minimum hedge ratio model

Country	Constant	Future's returns	Diagnostic	F-test
SP500	.00000449 (.000032)	.969760 (300.24)	R^2 = .966 DW= 2.209	41051.28
FTSE100	.00000401 (.000028)	.986590 (338.36)	R^2 = .973 DW= 2.182	52511.99
NIKKEI225	-.0000113 (.000046)	.950204 (251.967)	R^2 = .950 DW= 2.256	27325.44

Table 3: MCRR Estimates-GARCH Hedging Models for SP500

Days	Unhedged	Naïve Hedge	GARCH	GARH-ECM	GARCH-X	GARCH-BEKK	GARCH-GJR
A. Long cash and short futures:							
1	18.963	7.054	4.411	3.639	4.485	3.588	4.099
10	57.803	14.773	14.904	12.321	14.481	11.787	13.686
20	81.668	20.582	20.715	18.313	19.771	16.998	18.982
30	102.467	24.603	24.773	21.478	24.319	21.491	23.572
60	151.403	36.181	35.889	32.912	34.547	32.442	33.403
90	194.081	43.010	45.182	41.298	43.660	41.276	41.695
180	295.596	59.008	65.638	58.695	61.298	60.596	57.442
B. Short cash and long futures:							
1	22.504	4.802	4.893	4.729	4.811	4.342	4.752
10	67.741	14.517	249.740	14.221	14.645	13.665	15.257
20	98.641	20.881	21.056	19.327	20.773	19.727	21.417
30	119.313	25.929	25.655	25.749	25.949	24.205	25.949
60	172.175	36.314	36.606	36.559	36.054	34.482	36.948
90	218.328	45.444	43.747	45.099	46.829	44.171	44.405
180	312.605	67.081	63.332	65.414	65.814	61.412	63.253

Table 4: MCRR Estimates-GARCH Hedging Models for FTSE 100

Days	Unhedged	Naïve Hedge	GARCH	GARH-ECM	GARCH-X	GARCH-BEKK	GARCH-GJR
A. Long cash and short futures:							
1	83.973	15.833	16.279	16.578	57.447	13.992	14.952
10	262.175	53.397	54.226	56.031	178.795	46.262	49.840
20	374.122	81.953	85.206	86.234	259.484	67.819	76.334
30	473.162	106.259	103.836	106.537	340.802	82.326	102.146
60	735.222	160.202	158.581	164.168	499.242	116.852	153.508
90	910.980	194.519	199.975	201.029	645.510	146.583	195.111
180	1536.973	285.947	295.642	294.739	1027.968	213.484	316.040
B. Short cash and long futures:							
1	78.186	15.513	12.250	12.151	12.974	13.216	12.644
10	241.106	97.372	43.482	42.968	43.156	45.942	43.177
20	369.589	85.772	62.699	63.124	63.098	64.588	75.390
30	460.487	105.897	75.846	77.394	77.263	80.542	78.905
60	691.725	157.584	111.259	114.375	112.361	113.321	118.410
90	915.549	194.120	141.473	141.483	148.561	143.996	157.390
180	1383.869	283.633	219.717	227.691	218.552	207.230	236.610

Table 5: MCRR Estimates-GARCH Hedging Models for Nikkei S25

Days	Unhedged	Naïve Hedge	GARCH	GARH-ECM	GARCH-X	GARCH-BEKK	GARCH-GJR
A. Long cash and short futures:							
1	239.822	48.353	50.295	51.091	47.122	47.098	53.551
10	771.738	167.238	184.896	186.126	165.827	170.760	190.902
20	1179.075	251.738	281.408	270.975	268.370	261.713	286.438
30	1478.117	333.898	350.482	350.168	345.223	351.833	366.535
60	2251.929	535.715	557.208	557.956	532.205	536.463	573.116
90	2997.880	691.073	719.441	695.283	662.742	690.808	721.009
180	4985.781	1050.601	1074.120	1044.148	1019.737	1042.906	1120.024
B. Short cash and long futures:							
1	232.187	45.916	43.806	47.629	47.834	44.976	45.337
10	751.535	165.079	162.378	165.490	159.206	158.639	159.507
20	1125.307	250.954	243.319	261.290	246.057	247.058	236.203
30	1437.145	327.192	314.383	338.095	319.605	314.769	317.543
60	2273.518	513.993	491.885	522.496	508.566	499.641	497.952
90	2987.146	664.563	643.999	671.969	665.432	649.596	656.531
180	4757.972	999.812	965.209	1085.807	989.464	964.268	999.076