Circuit Breakers and Market Runs[1]

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Abstract

This paper analyzes whether the application of a "circuit breaker" to a financial market (i.e. a mechanism that interrupts trading for a predetermined period when the price moves beyond a predetermined level) reaches its intended goals of increased market stability and overall welfare. Our framework of analysis is a model in which investors can trade at several dates and might face a liquidity shock forcing them to sell immediately when the shock occurs. This setting potentially induces a "market run" where investors commonly sell merely out of fear other investors are selling and not because they have current liquidity needs. We show that the introduction of a sufficiently tightly-set circuit breaker within this setting successfully prevents this market run from occurring. Even more so, it could induce the socially optimal state (in which trading only takes place when it is motivated by liquidity needs) to arise. However, this desirable equilibrium can only be reached under particular economic conditions. When these conditions are not met, installing a circuit breaker might even lower social welfare as compared to a setting without a circuit breaker as it impedes socially desirable trades and stimulates socially undesirable trades.

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On May 6, 2010, at 2:32 p.m. a large mutual fund firm initiated an automated execution algorithm to aggressively sell a large index-futures position (valued at approximately $4.1 billion) as a hedge to an existing equity position. In the next few seconds, this action generated an unprecedented huge "hot-potato" volume effect exposing the fragility of current-days financial markets: still lacking sufficient demand from fundamental buyers or cross-market arbitrageurs, high frequency traders were rapidly buying and reselling contracts to each other. Having launched a vicious liquidity spiral, the initial local non-informational event soon triggered broader contagion effects causing equity markets to instantly dry up and indices to decrease by 5-6%. Twenty minutes later, however, these losses were in turn quickly recovered and the market had regained most of the drop. Quite aptly, this unprecedented sequence of events igniting a short-term liquidity crisis was labeled "flash crash".[1]

Since then, discussions at the regulatory and market level reemerged globally in order to assess the appropriateness of so-called "circuit breakers" in preventing or alleviating such liquidity crises and their undesired consequences on market volatility and overall financial stability. Circuit breakers were introduced after the global market crash of October 1987. Since then, they are a widely used technique in the stock market industry aiming at curbing and avoiding extreme price volatility and the resulting massive panic sell-offs. More specifically, circuit breakers are mechanisms that monitor the market continuously and trigger a trading halt as soon as the price of an individual security or of an index goes (or is bound to go) beyond a predetermined level. This halt may be temporary or, under extreme circumstances, close the market before the normal close of the trading session. In essence, circuit breakers were designed to slow down the market in the event of a sharp market movement, and thus prevent trading from occurring at prices far off fundamental values. As they artificially impede any panic trading from taking place, circuit breakers give traders the time to properly analyze the true nature of the underlying shock inducing the initial significant price change, and help the market revert to equilibrium. Evidently, this is a desirable feature when the underlying shock is liquidity-driven (and the volatility is transitory). In this case, the circuit breaker protects the market from liquidity traders' volatility-inducing trades, and protects the liquidity traders from trading losses that they will likely incur in times of poorly functioning markets. However, in case the underlying shock is information-driven (with fundamental volatility), circuit breakers cause prices to adjust slowly to new information which imposes costs on the economy.[2] In this case, a trading halt will merely postpone the inevitable and as prices are less informative while markets are closed and no one is certain of the new level, the uncertainty during the halt may

[1] See SEC (2010), Easley, Lopèz de Prado and O'Hara (2011), and Menkveld and Yueshen (2011) for an in-depth analysis.

[2] Stock prices accurately reflecting all information about firm's prospects indeed are essential for the proper allocation of available resources.

cause liquidity traders to panic generating additional unnecessary transitory volatility at reopening. Moreover, a further potential downside lies in the fact that circuit breakers induce uncertainty on the ability of traders to trade and thus accelerate price changes (Subrahmanyam (1994)), and thus increase transitory volatility which is exactly the reverse of what they are intended to do.

In the wake of the flash crash, more than two decades later than the 1987 crash, questions arose to what extent the design of these trading halt devices was still adequately tailored to function in the current high frequency trading times in which the trader population drastically altered (i.e., trading and liquidity provision became more computerized), trading became decentralized, and volumes and volatility have significantly increased (see e.g. Brogaard (2011) and Hendershott, Jones and Menkveld (2011)). To thoroughly investigate this important issue, right after the flash crash regulators and markets in the US set up a large-scale pilot experiment in which the existing rudimentary market-wide circuit breaker regime[3] has been adjusted and complemented by tailor-made narrowly set per-stock mechanisms.[4] In parallel, European regulators and markets are jointly examining which steps should be taken to move from the current fragmented circuit breaker regime - in which trading venues decide upon their individual circuit breaker rules, and see it as a competitive element to attract order flow - to a unified framework in which circuit breaker rules could function across markets when needed.[5]

This paper aims to contribute to the ongoing debate between regulators, market operators and market practitioners on the validity of circuit breakers. More specifically, our model allows to analyze the usefulness of installing a circuit breaker regime and yields insights on the optimal circuit breaker configuration - providing financial stability and allowing market participants to better manage their risks - for a range of market conditions. Our main starting point is a setting in which agents fail to coordinate their actions and trade massively although they should refrain from it. This setup is e.g. in line with evidence documented by Shiller (1987) in a survey conducted just after the crash of October 1987: investors did not respond to an information event in that

[3] As a reference, since their adoption in the US in October 1988, market-wide circuit breakers have only been triggered once on October 27, 1997 when the Dow Jones Industrial Average (DJIA) fell by 350 points by 2:35 p.m. This just triggered the standing circuit breaker which halted trading for one-half hour. In the 25 minutes following the reopening at 3:05 p.m., the DJIA dropped by an additional 200 points to trigger a second trading halt closing the market for the day.

[4] See www.sec.gov/answers/circuit.htm for detailed information.

[5] For instance, when the European Commission published its review of the Markets in Financial Instruments Directive (MiFID II), by codifying general rules on the installation of circuit breakers in regulated markets as well as mandating communications between trading venue operators it aims to better manage high-frequency trading. Biais and Woolley (2011) argue that this tailored cross-platform application of circuit breakers is essential to guarantee their effectiveness. Linking up to the flash crash, they indicate that since HFT algorithms arbitrage across markets, halting trades in the underlying while continuing trading in the derivative (or vice versa) can be very damaging.

crash, but anxiety and more generally psychological reasons triggered the massive price movements. We analyze how an optimally designed circuit breaker can prevent such a "market run", and thus prevent unnecessary trading losses and price movements. Our framework of analysis is the seminal model of bank runs developed by Diamond and Dybvig (1983). More specifically, in line with Bernardo and Welch (2004), we create a setting where agents know they might face urgent liquidity needs in the future (i.e., if they receive a liquidity shock at a later period, they have to sell immediately). If these agents expect the other agents to already sell within the current period (i.e., before the possible liquidity shock is realized), they will also opt to trade now instead of waiting until they have learned about their liquidity needs. Agents know that the market has a limited capacity to absorb large volumes instantly, and in their decision to trade they take into account the price drop induced by the sell orders of the other agents. As such, they sell without knowing whether they truly need to driven by the fear to trade at a worse price if they wait until the liquidity shock is realized, and as a consequence a market run could occur. Installing a circuit breaker within this setting limits the volume that can be traded in both trading periods: if excessive volumes are pushed into the market at one point in time, trading is interrupted within that trading round.

Our results indicate that the effect of the circuit breaker depends on how the price limit at which trading is interrupted is defined relative to (i) the price impact imposed to traders when they sell (which depends on the capacity of the market to absorb large volumes and on the risk of the asset) and (ii) the loss traders expect due to the possibility of a liquidity shock (which hinges on the probability of the liquidity shock and on the expected payoff of the asset).[6]

We distinguish four separate equilibrium cases in our analysis depending on the parameter regions where the price limit of the circuit breaker is set. First, if the installed circuit breaker is very lenient (i.e. the price limit is set at a very wide level), trading is never impeded and a circuit breaker is totally ineffective in preventing a market run from occurring. We label this the *"unrestricted equilibrium"*. Second, if the set price limit is tight relative to the price impact and if the expected loss of traders in case of a liquidity shock is small, the selling pressure in the early trading period is weak enough not to trigger the circuit breaker in that round. However, as only few traders liquidate early, the volume is large in case the liquidity shock realizes which pushes the price down and triggers the circuit breaker in the late trading period. In equilibrium, traders rationally anticipate that trading is halted if they suffer a liquidity shock and therefore might be induced to trade more in the early trading round as compared to the unrestricted case. We denote this case as the *"early-trading equilibrium"*. The third case

[6] Do note that the price impact directly affects the price level and thereby whether the circuit breaker is triggered or not. In turn, the expected loss exerts an indirect influence on the price level through the size of the volume pushed to the market as it determines the eagerness of traders to trade early.

that we highlight is one in which the price limit corresponding to the circuit breaker takes intermediate values and the expected loss related to a liquidity shock is large. In this case, many traders would like to trade early but the selling pressure prevents them from doing so as it triggers the circuit breaker. Thus, trading is halted and postponed to the next trading round. As such, within this case, traders only realize trades if these are motivated by liquidity reasons. We label this case the *"late-trading equilibrium"*. Finally, within our fourth case the circuit breaker is set very tightly and the expected loss due to the liquidity shock is large. Trading is halted in both trading periods within this *"no-trading equilibrium"*.

Overall, within this model, there is a flavor of two types of outcomes: a "good" one in which agents delay (voluntarily or not) trading until the realization of the liquidity shock (and therefore trade only when it is socially desirable), and a "bad" one in which most or all agents trade early without knowing what their future liquidity needs will be (thereby incurring socially undesirable trading costs). In the "late-trading equilibrium", the circuit breaker exogenously forces agents to delay trading until they are aware of their true liquidity needs. In fact, this mechanism resolves the existing coordination failure among traders and leads to the highest possible level of social welfare. However, the implementation of this equilibrium is linked to specific economic conditions and is therefore not always realizable. When these conditions are not met (and thus when one of the other three equilibria is played), some trade-offs have to be accounted for when analyzing welfare. In particular, a restrictive circuit breaker may actually prevent the realization of socially desirable trades, and stimulate the occurrence of socially undesirable trades.

This paper relates and contributes to several strands within the existing academic literature.

First of all, we aim to provide new insights to the theoretical literature on trading halts which dates back to the period right after the 1987 crash.[7] Published theoretical work presents analyses of the losses and benefits for different trader groups associated with the existence and the triggering of a circuit breaker. Greenwald and Stein (1991) and Kodres and O'Brien (1994) argue that a circuit breaker might lead to more liquidity provision as it incentivizes additional value-motivated traders to enter the market. In contrast, Subrahmanyam (1994) highlights negative circuit-breaker-related consequences by demonstrating that a circuit breaker might induce agents to trade large volumes

[7] Most of the important contributions were published in the early nineties, since then the topic received very little academic attention. Noteworthy, theoretical models could be argued to be a valid and necessary tool to properly address this topic, as empirical work features some major caveats as proper event studies are difficult to design because of (i) small samples, and (ii) the lack of an appropriate counter-factual (i.e., what would have happened if there was no circuit breaker in place). See Kyle (1988), Moser (1990), and Harris (1998) for detailed discussions on the theoretical analyses of circuit breakers.

earlier out of fear that the circuit breaker might be triggered before they can submit their orders. In such a case, the circuit breaker leads to higher transitory stock price volatility. In turn, Subrahmanyam (1995) suggests randomizing trading halts as a possible way to alleviate such a behavior.[8] Furthermore, Morris (1990) analyzes the case of two markets trading the same asset with only one of these markets having a circuit breaker installed. He shows that within this setting order flow will merely divert to the other market during a trading halt, and proposes cross-market circuit breaker regime as a cure for the resulting increase in volatility. As mentioned above, this paper addresses the question whether circuit breakers are desirable from an overall social welfare perspective. More specifically, the welfare effects of circuit breakers will be investigated from different angles, and their role in preventing market failures will be analyzed in depth. Whereas this research question seems relevant and self-evident, to our knowledge it has never been addressed before in the academic literature on trading halts.

Second, our paper contributes to the current literature on high frequency trading (HFT) and market stability. Ex ante, it is not precisely clear whether the augmentation of fast computerized trading increases or lowers market stability. On the one hand, trading algorithms pre-programmed by humans might exhibit inferior performance in providing market stability as compared to humans when market conditions exceed the circumstances they are programmed to function in. According to this argument, human flexibility is seen as an asset in decision-making when times get rough. In contrast, this benevolent flexibility may also have a reverse effect. Overreaction to new and uncertain market conditions may lead the market to worse states than mere rational algo traders would.[9] Several recent theoretical papers have shed light on this trade-off. Biais, Hombert and Weill (2010) argue human traders exhibit limited cognitive abilities to manually process the vast amounts of information available these days, which could prevent them from realizing their trading gains. Installing trading algorithms could compensate this limited rationality and thus improve market liquidity as they mitigate existing market imperfections.[10] In turn, analyzing "equilibrium high frequency trading", Biais, Foucault and Moinas (2011) focus on the implicit adverse selection effect corresponding to a state where a subset of market participants is entitled to early and privileged access to relevant

[8] The existing empirical evidence also produces mixed results. Gerety and Mulherin (1992) find evidence consistent with the idea that circuit breakers can destabilize the market, and Lee et al. (1994) show that the introduction of circuit breakers has no significant downward effect on volatility, and may even increase volatility. On the bright side, Lauterbach and Ben Zion (1993) find evidence indicating circuit breakers may induce a reduction in order imbalances. Further, Christie et al. (2002) find mixed results on their sample of trading halts on Nasdaq: in the case of short intraday halts volatility increases after the halt, but in the case of a reopening the next day, volatility effects are dampened.

[9] See Angel and McCabe (2010), Biais and Woolley (2011), Hendershott (2011), and Menkveld (2011a) for a further discussion of this tradeoff.

[10] This result is in line with existing empirical evidence by Brogaard (2011), Hendershott and Riordan (2011), Menkveld (2011b) and Kirilenko, Kyle, Samadi, and Tuzun (2011).

information and has the ability to trade on it before others.[11] Their findings indicate that HFT algorithms can increase the trading gains realized in the market (by helping traders to find counterparties), but also that "slow humans" indeed bear the downside of the implicit informational edge in this non-level playing field and incur the adverse selection cost (preventing the realization of these gains from trade).[12] The current paper complements this strand of the literature in that it in-depth analyzes to what extent one of the widely acclaimed and applied measures to reduce the undesired effects of HFT trading and ensure market stability (i.e. circuit breakers) may or may not be adequate in reaching this set goal. To our knowledge, this paper provides the first model ever to analyze the role of circuit breakers in solving existing coordination failures between traders and thus prevent financial markets from fully breaking down.

The remainder of this paper is structured as follows. Section 1 introduces the setup of our model. Next, Section 2 presents the analysis of the market equilibrium in the absence and in the presence of circuit breakers. Furthermore, Section 3 analyzes how installing circuit breakers affects social welfare. Finally, Section 4 concludes.

1 Setup

We consider an economy with three dates ($t = 0, 1, 2$), two assets (a risk-free bond and a risky asset), and two types of agents (risk neutral traders and a risk averse liquidity supplier). The traders are able to trade in $t = 0$ and in $t = 1$. At $t = 2$ the value of the assets is realized. The risk free bond is in infinitely elastic supply and pays off 1\$. In turn, the risky asset has a payoff $\widetilde{Z}$ which is normally distributed with mean μ and variance σ^2. The entire supply of the risky asset is held by a mass 1 of traders: each trader holds one share. With probability s, all traders are hit by a liquidity shock at $t = 1$ and are obliged to sell their single-unit holding at that date. If they do not sell (or are not able to sell), holding the asset becomes worthless and their utility is zero (see e.g. Bernardo and Welch (2004) for a similar setting). If the traders are not hit by the liquidity shock, they receive the final payoff of the asset in $t = 2$. Traders also have the possibility to trade at $t = 0$. When the traders sell at $t = 0$ and/or $t = 1$, they submit their order flow to the same liquidity supplier. Traders are price takers and decide whether to sell their share such as to maximize their expected payoff.

Before the early trading round, the liquidity supplier (or market maker) does not have

[11] HFTs obtain these advantages and minimize their trading latency in return for an investment in costly informational processing equipment and co-location charges set by exchanges.

[12] Empirical work such as Brogaard (2011), Hendershott and Riordan (2011) and Kirilenko, Kyle, Samadi, and Tuzun (2011) indeed indicate the informational content of HFT orders is greater. Moreover, Jovanovic and Menkveld (2010) indicate the entry of an HFT on the Dutch market implied a 13% decrease in trading volume, which might be induced by the reduction in market participation by slow human traders.

an inventory in the risky asset. At each trading date, he can choose between two actions. He can participate in the market and absorb the trading volume submitted by traders at a particular price p_t with $t = 0, 1$. In that case he modifies his inventory position, knowing he cannot rebalance his inventory after or during the trading rounds. The market maker can also opt to stay out of the market and keep the inventory level constant. We assume that the market maker represents a competitive industry of liquidity suppliers. Therefore, the price at which the order flow is absorbed is such that the market maker is indifferent between trading and not trading. Since he is risk averse, the price offered by the market maker is below the expected value of the asset and decreases the higher his inventory position becomes. The market maker has a negative exponential utility:

$$u(\widetilde{W}) = -e^{-\gamma \widetilde{W}} \tag{1}$$

with $\widetilde{W}$ being the wealth in $t = 2$ and γ the risk aversion coefficient. The market maker is assumed to behave myopically. They do not take into consideration future trading possibilities when they set the price in $t = 0$.

Next, a circuit breaker is added to this market setting which limits trading in case it is triggered. In particular, when the price difference between two subsequent transactions becomes too large, trading is stopped immediately in the current period and is only relaunched at the next date.[13] Thus, the following rule defines the circuit breaker configuration:

- Trading occurs at date t if $p_{t-1} - p_t \leq \Delta$;
- Trading is interrupted at date t if $p_{t-1} - p_t > \Delta$.

with Δ a commonly-known threshold set by the exchange, and constant over time. The value of the asset before trading takes place in $t = 0$, is assumed to be its expected value μ. Since we consider only one side of the market, i.e. traders possibly selling, the circuit breaker is only defined relative to price decreases.

2 Market Equilibrium

2.1 Price-Setting by Market Maker

As a first step, we analyze how the market maker sets prices when trading is unrestricted (i.e. with no active circuit breaker as in Bernardo and Welch (2004)). Consider the

[13] This setup corresponds to actual market practice, where the arrival of an order potentially pushing the transaction price beyond a pre-set reference price triggers the circuit breaker and instantly halts all trading (including the order triggering the circuit breaker).

market maker's decision in the initial trading period (at $t = 0$) and assume that a fraction α of traders sells the asset. If the market maker decides to abstain from absorbing any volume, his inventory does not change and the final wealth he expects to have is W_0. In contrast, if the market maker buys the shares sold by the traders, his inventory increases by α shares and his expected final wealth $\widetilde{W_2}$ is $W_0 + \alpha\left(\widetilde{Z} - p_0\right)$. Then, the share price $p_0(\alpha)$ set by the competitive market maker should make him indifferent between these two actions:

$$E\left[-e^{-\gamma W_0}\right] = E\left[-e^{-\gamma\left[W_0+\alpha\left(\widetilde{Z}-p_0\right)\right]}\right] \tag{2}$$

which, with CARA utility, is equivalent to:

$$E\left[W_0 + \alpha\left(\widetilde{Z} - p_0\right)\right] - \frac{\gamma}{2}Var\left[W_0 + \alpha\left(\widetilde{Z} - p_0\right)\right] = W_0$$

$$\Leftrightarrow W_0 + \alpha\left(\mu - p_0\right) - \frac{\gamma}{2}\alpha^2\sigma^2 = W_0$$

and yields a price equal to:

$$p_0(\alpha) = \mu - \frac{\gamma}{2}\sigma^2\alpha \tag{3}$$

At the next trading period $t = 1$ the market maker already has an inventory of α shares. If a liquidity shock occurs, he increases this inventory by $(1 - \alpha)$ shares when participating in the market. In this case, he expects final wealth to increase to: $W_0 + \alpha\left(\widetilde{Z} - p_0\right) + (1 - \alpha)\left(\widetilde{Z} - p_1\right)$. In contrast, if he opts to stay out of the market, he keeps the inventory α with corresponding wealth $W_0 + \alpha\left(\widetilde{Z} - p_0\right)$. Thus, the price $p_1((1 - \alpha); \alpha)$ set by the market maker at $t = 1$ given his starting inventory α must compensate the additional risk he accepts to take by buying the assets, and make him indifferent between these two actions:

$$E\left[u\left(W_0 + \alpha\left(\widetilde{Z} - p_0\right)\right)\right] = E\left[u\left(W_0 + \alpha\left(\widetilde{Z} - p_0\right) + (1 - \alpha)\left(\widetilde{Z} - p_1\right)\right)\right] \tag{4}$$

This implies a price:

$$p_1((1 - \alpha); \alpha) = \mu - \frac{\gamma}{2}\sigma^2(1 + \alpha) \tag{5}$$

Noteworthy, the price at the late trading round not only decreases with the risk level of the asset σ^2 and with the market maker's risk aversion γ, but also with the size of the volume traded in the previous trading round α. Thus, a smaller trading volume at $t = 0$ (or equivalently a higher trading volume at $t = 1$) leads to a lower price impact in $t = 1$. Although this last effect seems counter-intuitive, it directly stems from the different consequences an increase in the inventory has on the expected payoff and on the variance of the holding. Although the market maker ends up with a volume of 1, he imposes a price impact that is larger than $\frac{\gamma}{2}\sigma^2$ by the factor of $(1 + \alpha)$. The associated price decrease allows the market maker to compensate the additional risk he takes by absorbing the additional volume in order to make him indifferent to the previous inventory position. Thus, in equilibrium, the price $p_1((1 - \alpha); \alpha)$ is such that the increase in the expected payoff compensates the increase of the disutility due to a larger inventory:

$$(1 - \alpha)(\mu - p_1) = \frac{\gamma}{2}\sigma^2(1 - \alpha^2) \tag{6}$$

where the LHS is the increase in the expected payoff if the market maker absorbs the additional volume and the RHS is the increase in the disutility. While the increase in the expected payoff is linear in the volume α, the increase in the disutility is quadratic in α. Thus, when the traded volume in $t = 1$ becomes smaller (which is equivalent to a higher α), the decrease of the additional expected payoff is larger than the decrease of the additional disutility due to risk. To compensate the higher disutility, the market maker has to reduce the price more with higher α. The factor by which the price is reduced $(1 + \alpha)$ corresponds to the factor by which the decrease in the variance of the holding is smaller than the decrease in the expected payoff.[14]

2.2 Circuit Breaker Thresholds

In a next step, we introduce a circuit breaker to this setting. This mechanism is triggered when the transaction price at t decreases substantially with respect to the previous transaction price. In the early trading round (at $t = 0$), trading is interrupted when the trading volume is "too high" (i.e. when α is too large since a high volume increases the inventory of the market maker and pushes the price down). More precisely, the circuit breaker is triggered if $\mu - p_0(\alpha) > \Delta$, which is equivalent to $\frac{\gamma}{2}\sigma^2\alpha > \Delta$. Thus, for a given level of the circuit breaker level Δ the threshold of the volume size up from which trading is interrupted is:

[14] More specifically: $(1 - \alpha)(\mu - p_1) = \frac{\gamma}{2}\sigma^2(1 - \alpha)(1 + \alpha)$ then holds.

$$\widetilde{\alpha} = \frac{2\Delta}{\gamma\sigma^2} \tag{7}$$

If the traded volume at $t = 0$ is smaller than or equal to this level (i.e. $\alpha \leq \widetilde{\alpha}$) the circuit breaker is never triggered and trading always takes place. In contrast, if the volume is larger than this threshold (i.e. $\alpha > \widetilde{\alpha}$), trading is always interrupted and traders cannot sell their shares in $t = 0$. Since we consider a competitive market maker, the price he offers $p_0(\alpha)$ is the highest price he is willing to pay. At a higher price, the trading interruption could be avoided but the market maker would be better off staying out of the market. Thus, the market maker never has an incentive to set a price which could prevent the trading interruption. As such, only the action chosen by traders α determines whether the circuit breaker is triggered or not.

In the late trading round at $t = 1$, the conditions under which the circuit breaker is triggered depend on whether traders were able to trade in the previous trading round or not. We consider these two distinct cases separately:

- In the first case, trading took place in the early trading round such that the market maker now has a positive inventory in the asset and the newly-set reference price is $p_0(\alpha)$. It follows that the circuit breaker is triggered in $t = 1$ if the price difference becomes too large, i.e. $p_0(\alpha) - p_1((1-\alpha)\,;\alpha) > \Delta$. This occurs when the circuit breaker level Δ is equal to or smaller than the risk premium required by the market maker on the entire volume of 1:

$$\frac{\gamma}{2}\sigma^2 > \Delta \tag{8}$$

- In the second case, the circuit breaker has been triggered in the early trading round such that trading was interrupted. As a result, the market maker has kept his inventory at the initial zero level and the reference price for the circuit breaker is now $\mu - \Delta$ (i.e. the attained circuit breaker level of the previous trading period). The market reopens for the late trading round (at $t = 1$), and the circuit breaker is triggered if the transaction price for the entire volume is too low relative to this reference price, i.e. $\mu - \Delta - p_1(1;0) > \Delta$. This is the case if the circuit breaker level is equal to or smaller than half of the risk premium required on the entire volume:

$$\frac{\gamma}{4}\sigma^2 > \Delta \tag{9}$$

The computation of both these thresholds for the late trading round relies on the assumptions that (i) the market maker never sets a higher price in equilibrium and (ii) the traders all sell their shares if they are hit by a liquidity shock. In fact, the first assumption always holds since the competitive market maker offers the highest price he is willing to pay, as in the early trading round. Moreover, the second assumption will turn out to correspond to equilibrium behavior by traders, as we will see later on in this section. Intuitively, if traders suffer a liquidity shock, they do not have any utility gain in keeping the asset. Therefore, none of them will be willing to refrain from trading in order to let others trade. Furthermore, the possibility to trade at $t = 1$ is determined completely exogenously and does not depend on any strategic choices of the traders. Therefore the circuit breaker levels at which trading is interrupted are fully determined by the parameters of the model: i.e. the risk aversion of the market maker γ and the risk level of the asset σ^2. For a given trading volume in the late trading period $1 - \alpha$ only these parameters determine the price impact and thus whether or not the circuit breaker is triggered. Interestingly, the computed thresholds indicate that the parameter regions at which trading can occur in $t = 1$ differ depending on whether or not trading took place in the previous trading period. If traders already traded at $t = 0$, trading in the late round can only occur if the circuit breaker level Δ is larger than in the case in which traders did not trade in $t = 0$ because trading was interrupted. This is due to the difference in the reference prices and the difference in the transaction prices at $t = 1$ between both cases. In the first case with trading in $t = 0$, the reference price is higher and the transaction price is lower than in the second case. Thus, when trading took place in the initial trading period, the circuit breaker level Δ must be larger to also allow trading in the late trading period.

Overall, this analysis yielding the exogenous circuit breaker thresholds already shows two general beneficial effects of a circuit breaker: when the selling pressure becomes too large in the early period, a circuit breaker (i) interrupts trading so as to avoid a large number of socially undesirable trades corresponding to a market run, and (ii) subsequently permits traders to trade in a larger parameter range at $t = 1$ when they need to trade because of the shock. Furthermore, do note traders can influence the trading possibilities in $t = 1$ by choosing α in $t = 0$, but only to a limited extent. Only for a middle range of Δ trading in $t = 1$ depends on whether traders traded at the previous date. For very tight and very lenient levels of Δ, trading in $t = 1$ is completely independent of the actions at $t = 0$. We summarize these results in the following lemma.

Lemma 1 *Assuming that a liquidity shock has occurred at $t = 1$:*

- *When $\Delta > \frac{\gamma}{2}\sigma^2$: trading always takes place at $t = 1$, regardless of whether or not trading occurred at $t = 0$;*

- *When $\frac{\gamma}{4}\sigma^2 < \Delta \leq \frac{\gamma}{2}\sigma^2$: trading only takes place at $t = 1$ if trading was halted by the circuit breaker at $t = 0$, otherwise it is halted by the circuit breaker;*

- *When $\Delta \leq \frac{\gamma}{4}\sigma^2$: trading is always halted by the circuit breaker at $t = 1$, regardless of whether or not trading occurred at $t = 0$.*

These results indicate that a very tight circuit breaker level always prevents traders from trading when they are hit by a liquidity shock at $t = 1$ (i.e. when trading is socially desirable). As such, the existence of a circuit breaker might in fact push them into socially undesirable trading at $t = 0$, out of fear of a liquidity shock in the next period. However, traders will not be able to engage in a run because the same tight limit applies to trading at $t = 0$. Indeed, for trading to take place in $t = 0$, α has to be smaller than 0.5. In turn, if the circuit breaker limit takes an intermediate level, the socially desirable result can be reached in which traders only trade when they are hit by the shock. However, this intermediate case also encompasses the socially less desirable case in which traders only trade in the early period and thereby prevent themselves from being able to trade in case of a shock. Finally, a very lenient circuit breaker limit never hinders trading at $t = 1$ in case of a shock. However, the same loose limit evidently also applies for trading in the early period. Thus, under this regime, traders might be able to run or to trade large quantities when these are socially undesirable. In that sense, the circuit breaker might prove to be useless in preventing socially undesirable trades.

2.3 Equilibrium Behavior

Now that we have derived the two exogenous thresholds for the setting including a circuit breaker, let us focus on the optimal trading decisions (i.e. the determination of α) within the three parameter zones corresponding to these thresholds.

Consider an individual trader who deliberates on selling his share. If he anticipates that α other traders will also sell their share to the market maker at $t = 0$, he will expect to receive the price $p_0(\alpha)$ for his share assuming that the circuit breaker will not be triggered. Since the circuit breaker's threshold level at $t = 0$ (i.e. $\widetilde{\alpha}$) only depends on exogenous parameters, the trader knows with certainty whether a conjectured volume induces an interruption in trading or not. In turn, if the trader opts to wait until $t = 1$, he will either be forced to sell his share with probability s (if trading can take place at that date), or he can hold on to the asset until $t = 2$ to receive the expected value μ with probability $1 - s$. Now, define the indicator variable I_t (with $t = 0, 1$) as a variable taking the value 1 if trading takes place in period t and 0 if the circuit breaker is triggered in period t. At $t = 1$, trading only hinges on exogenous variables and therefore I_1 is exogenously determined. In turn, at $t = 0$ trading depends on the volume submitted to

the market maker: $I_0(\alpha)$. Do note the transaction price at $t = 1$ depends on α but also on $I_0(\alpha)$. If trading was interrupted at $t = 0$ because of a too large order flow α, when trading occurs in $t = 1$ the complete volume is sold as if α had been zero in the early trading round.

The trader sells his share for a conjectured price $p_0(\alpha)$ only if the price he receives is larger than the expected payoff when waiting until the next trading round. This trade-off is captured by the function $F(\alpha)$, which is the expected net benefit of selling shares at $t = 0$:

$$F(\alpha) = p_0(\alpha).I_0(\alpha) - s.p_1(\alpha, I_0(\alpha)).I_1 - (1 - s).\mu \tag{10}$$

This function differs from Bernardo and Welch (2004) to the extent that it integrates the possibility of a trading interruption. In particular, the transaction price at $t = 1$ now features a different form than in the unrestricted trading benchmark case since it hinges on whether trading took place at $t = 0$ or not. As long as the number of traders trading at $t = 0$ is small enough not to trigger the circuit breaker, the transaction price at $t = 1$ decreases with α as in Bernardo and Welch (2004). However, if trading was interrupted at $t = 0$, the price at $t = 1$ jumps up to the level that corresponds to the situation in which the entire trading volume is sold at $t = 1$: $p_1(\alpha > \widetilde{\alpha}, 0) = p_1(0, 1)$.

The strategies chosen by traders depend on the sign of this function. The case in which all traders wait until $t = 1$ (i.e., $\alpha^* = 0$) is a pure strategy Nash equilibrium iff $F(0) \leq 0$. In turn, the case in which all traders sell at $t = 0$ (i.e. $\alpha^* = 1$) is a pure strategy Nash equilibrium iff $F(1) \geq 0$. Finally, a fraction $\alpha^* \in (0, 1)$ of traders selling is a mixed strategy Nash equilibrium iff $F(\alpha^*) = 0$. As in Bernardo and Welch (2004), $\alpha^* = 0$ is never a Nash equilibrium. If nobody sells at $t = 0$, the inventory of the market maker remains at a zero level. Thus, if a single trader tenders his share, he would obtain the price without price impact μ, and moreover this trader knows that the circuit breaker will not be triggered in $t = 0$. As such, this trader has a costless insurance against tomorrow's liquidation risk and he will always deviate. Formally, the function $F(0)$ is always positive and jumps to a higher level when I_1 is equal to zero:

$$F(0) = \mu.1 - s.p_1(0, 1).I_1 - (1 - s).\mu > 0 \text{ for } I_1 = 0, 1 \tag{11}$$

Traders can perfectly anticipate in which of the three parameter regions described in Lemma 1 they will be at $t = 1$, since all parameters are common knowledge in the entire game. They take this knowledge into account when deciding upon whether or not to trade at $t = 0$. In the following subsections, we will consider each of the delineated

parameter ranges in turn, and analyze equilibrium behavior keeping the functional form $F(\alpha)$ mentioned in Equation (10) in mind.

2.3.1 First case: $\Delta > \frac{\gamma}{2}\sigma^2$

Within this parameter range traders know they can always trade at $t = 1$, regardless of their decision at $t = 0$ (see Lemma 1), and thus $I_1 = 1$. Moreover, the volume threshold triggering the circuit breaker within $t = 0$ is larger than one in this parameter region (i.e. $\widetilde{\alpha} > 1$). As such, trading takes place even if all traders sell their share at $t = 0$ (i.e. $I_0(\alpha^*) = 1$ for all $\alpha^* \in (0, 1)$). Thus, the circuit breaker is configured so leniently that it never impedes trading, and hence never prevents a market run. This unrestricted situation corresponds to the equilibrium derived in Bernardo and Welch (2004). Additional traders sell their shares at $t = 0$ as long as their expected profit, given the beliefs on the number of entering traders, is weakly positive:

$$F(\alpha^*) = p_0(\alpha^*) - s.p_1(\alpha^*) - (1 - s).\mu \geq 0 \tag{12}$$

We summarize traders' equilibrium behavior within the parameter range $\Delta > \frac{\gamma}{2}\sigma^2$ in the following proposition:

Proposition 1 *If $\Delta > \frac{\gamma}{2}\sigma^2$ trading is never halted. At $t = 0$, traders play the following strategies: if $s \geq \frac{1}{2}$ all traders sell their share (i.e. $\alpha^* = 1$) and there is a market run, if $s < \frac{1}{2}$ only a fraction $\alpha^* = \frac{s}{1-s}$ of traders sell their share. In the case of a liquidity shock at $t = 1$, all other traders sell.*

Proof. See Appendix. ■

As pointed out in Bernardo and Welch (2004), the trading volume is convex in the probability of a shock s: if traders expect that only a few other traders sell at $t = 0$, they prefer to wait, whereas if traders expect many of the others to sell, they also sell at $t = 0$ and the number of traders selling rises faster than the probability of a shock. This implies that the transaction prices at both dates are decreasing and concave in s and that the price volatility measured by the relative price difference between $t = 1$ and $t = 0$ is increasing and convex in s.

2.3.2 Second case: $\frac{\gamma}{2}\sigma^2 \geq \Delta > \frac{\gamma}{4}\sigma^2$

In this second case, traders know that when trading took place at $t = 0$, it will be halted in the next trading round in the case of a liquidity shock. However, if they submit a large enough order flow to trigger the circuit breaker at $t = 0$, they will be able to trade

in the next round in case of a liquidity shock (see Lemma 1). Put formally, if α^* is equal to or smaller than the threshold $\widetilde{\alpha}$, trading takes place at $t = 0$ so that $I_0(\alpha^* \leq \widetilde{\alpha}) = 1$ and $I_1 = 0$. In the opposite case: $I_0(\alpha^* > \widetilde{\alpha}) = 0$ and $I_1 = 1$.

Now, consider the former case first and assume that $\alpha^* \leq \widetilde{\alpha}$. In this parameter range we know that $\widetilde{\alpha} < 1$, thus this assumption implies that $\alpha^* < 1$. The equilibrium trading volume at $t = 0$ is determined by the following equality:

$$F(\alpha^*) = p_0(\alpha^*) - (1 - s).\mu = 0 \tag{13}$$

The trading volume α^* making traders indifferent between trading in $t = 0$ and waiting until the payoff is realized depends on the size of the price impact at $t = 0$ and the size of the loss in case of a shock at $t = 1$ (see proof of Proposition 2):

$$\alpha^* = \frac{2s\mu}{\gamma\sigma^2} \tag{14}$$

A higher price impact $\gamma\sigma^2$ induces fewer traders to sell at $t = 0$ and to accept a loss in case of a shock at $t = 1$. In turn, the larger the loss is when the liquidity shock occurs $s\mu$, the higher is the number of traders who are induced to anticipate the sale of their asset.

In contrast to the previous case, the trading volume here increases linearly with the probability of a shock s. As such, there no longer is an accelerator effect active. In fact, do note the accelerator effect evidenced in Bernardo and Welch (2004) and in the unrestricted case stems from the fact that both the trading loss if a trader trades at $t = 0$ and the trading loss if he trades at $t = 1$ increase in the early period's volume α. In that unrestricted equilibrium, the number of traders selling is such that the two losses are equal, or:

$$-\frac{\gamma}{2}\sigma^2\alpha = -s\frac{\gamma}{2}\sigma^2(1 + \alpha) \tag{15}$$

where the RHS represents the trading loss in $t = 1$ and the LHS the loss in $t = 0$. When a liquidity shock becomes more likely, the loss at $t = 1$ becomes larger not only because the risk premium is larger in expectation ($s\frac{\gamma}{2}\sigma^2$) but also because the effect of α on the loss becomes more important. It is the combination of these two effects that leads to an increasing and convex relationship between the equilibrium trading volume and the likelihood of a liquidity shock in the unrestricted case. But in the restricted case - where the circuit breaker takes intermediate values - the trading loss incurred at

$t = 1$ if traders do not sell at $t = 0$ is linear in s since there is no trading at $t = 1$. The equilibrium condition equalizing the trading losses is:

$$-\frac{\gamma}{2}\sigma^2\alpha = -s\mu. \tag{16}$$

Since α does not affect the RHS, a higher likelihood of the liquidity shock has no other effect than increasing the loss at $t = 1$ linearly which translates into a linear increase of α^*.

So far in this subsection, all results were derived and discussed under the assumption that α^* is small enough to allow trading at $t = 0$. Evidently, this is only true if α^* is smaller than the threshold $\widetilde{\alpha}$ which is equivalent to the following condition given the parameter region analyzed in this second case:

$$s\mu \leq \Delta \tag{17}$$

Within the analyzed parameter region traders know that they will not be able to trade in $t = 1$ if they trade in $t = 0$. Therefore, the total amount of shares traded in $t = 0$ increases with the loss traders suffer if they are hit by the liquidity shock in $t = 1$, $s\mu$. Thus, for trading to be possible, the circuit breaker rule Δ must be defined larger than the potential loss related to the liquidity shock and the impossibility to trade in that case, which is precisely what induces traders to trade more in $t = 0$.

Next, if the traders know that a number α^* would like to trade (i.e. $F(\alpha^*) = 0$) but that trading cannot take place because the price impact would be so large that the circuit breaker is triggered (i.e. $\alpha^* > \widetilde{\alpha}$), they can either restrict themselves or submit orders knowing that trading is then necessarily postponed to the next date. As already highlighted above, it appears that no trading cannot be an equilibrium because a deviating trader can trade without price impact at $t = 0$. Using similar arguments, it could be proven that restricting to any volume strictly smaller than $\widetilde{\alpha}$ is not deviation-proof (as there always are trading gains to be made and no subset of traders can be restricted to trade), such that the circuit breaker is always triggered at $t = 0$. As a consequence, the trading strategy of traders is always:

$$\alpha^* = \frac{2s\mu}{\gamma\sigma^2} \tag{18}$$

regardless of whether they trigger the circuit breaker or not. If the threshold at which the circuit breaker is triggered is large enough, α^* traders trade at $t = 0$ and those who

do not sell in the early trading round can obtain the expected payoff μ at $t = 2$ if they are not hit by a liquidity shock. In turn, if the circuit breaker is set quite tightly, trading at α^* is always halted. In that case traders can trade in $t = 1$ if they suffer a liquidity shock.

We summarize traders' equilibrium behavior within the analyzed parameter range in the following proposition:

Proposition 2 *If $\frac{\gamma}{2}\sigma^2 \geq \Delta > \frac{\gamma}{4}\sigma^2$, trading can only take place within one of both trading periods:*

- *If $s\mu < \Delta \leq \frac{\gamma}{2}\sigma^2$, trading only takes place at $t = 0$ and $\alpha^* = \frac{2s\mu}{\gamma\sigma^2}$. Trading is always halted at $t = 1$.*
- *If $\frac{\gamma}{4}\sigma^2 < \Delta \leq s\mu$, trading is halted at $t = 0$. Trading only takes place at $t = 1$ in the case of a liquidity shock.*

Proof. See Appendix. ■

Within this parameter range, the circuit breaker avoids a market run to happen since trading at $t = 0$ is always smaller than 1. Thus, by avoiding large trading volumes at $t = 0$ it prevents socially undesirable trades from occurring, which is the positive side of the circuit breaker. However, if the selling pressure at $t = 0$ is not large (e.g. due to a low probability of a liquidity shock s), the existence of the circuit breaker has two negative effects. First, it might lead to a higher (socially undesirable) trading volume at $t = 0$ than would be the case in the unrestricted case, as traders take into account the impossibility to trade at $t = 1$. Second, the circuit breaker hinders traders to trade at $t = 1$ if they need to because of the liquidity shock. As such, it impedes socially desirable trades from occurring.

2.3.3 Third case: $\Delta \leq \frac{\gamma}{4}\sigma^2$

In this third case, traders know that they will never trade at $t = 1$ (see Lemma 1). Thus, the equilibrium number of traders trading at $t = 0$ is determined as in the previous case, assuming that trading takes place at $t = 0$:

$$F(\alpha^*) = p_0(\alpha^*) - (1 - s).\mu = 0 \Leftrightarrow \alpha^* = \frac{2s\mu}{\gamma\sigma^2} \tag{19}$$

However, the circuit breaker level Δ is tighter than in the previous case and therefore, the traded volume must be smaller than in the previous case in order not to trigger the

circuit breaker. Some higher volume levels that can be traded in the previous case cannot be traded here (see proof of Proposition 3).

As in the previous two cases, in case α^* triggers the circuit breaker no trader will restrict himself from selling his share for the benefit of the others. Due to this coordination failure among traders, trading is never restricted in equilibrium and traders always submit a total order flow of $\alpha^* = \frac{2s\mu}{\gamma\sigma^2}$, even when they know that trading will be halted.

Proposition 3 *Trading is always halted at $t = 1$.*

- *If $s\mu < \Delta \leq \frac{\gamma}{4}\sigma^2$, a fraction $\alpha^* = \frac{2s\mu}{\gamma\sigma^2}$ of traders trades at $t = 0$.*
- *If $\Delta < s\mu \leq \frac{\gamma}{4}\sigma^2$, trading is also halted at $t = 0$.*

Proof. See Appendix. ■

Thus, within this case, the circuit breaker is defined so tightly that trading might never occur, preventing socially undesirable trades but also socially desirable trades from occurring. Although market runs are always avoided, this case allows only for socially undesirable trading in $t = 0$ if anything.

2.4 Discussion of the Different Equilibria

We finalize this market equilibrium section by distilling four unique equilibria differing in the way trading strategies are affecting the circuit breaker rule across both trading rounds. These four equilibria are depicted by differently-colored areas in Figure 1.

First of all, the yellow area represents the equilibrium in which the price limit of the circuit breaker is defined in such a lenient way that trading is never impeded. This zone perfectly coincides with the first case mentioned within the previous subsection (i.e. with $\Delta > \frac{\gamma}{2}\sigma^2$), we label it the *"unrestricted equilibrium"* as the presence of the circuit breaker does not affect the trading possibilities and the equilibrium choices of traders. More specifically, the realization of this equilibrium does not depend on the probability of the liquidity shock s in the sense that trading always takes place for any value of s. However, the number of traders selling their asset depends on the size of s since a market run occurs only if a future liquidity shock has a likelihood of more than 50%. Do note this unrestricted equilibrium perfectly corresponds to the Bernardo and Welch (2004) outcome.

In turn, when the circuit breaker is more restrictive, we can identify three other cases in which trading is halted in at least one trading round due to the triggering of the circuit breaker: either trading takes place in the early trading round only and is interrupted in

the late round, or trading is only possible in the late trading round because it is halted in the early one, or trading is interrupted in both trading rounds. In these three cases, the circuit breaker has two effects: not only does it influence the trading possibilities of traders, but it also affects the number of traders trading in equilibrium (as compared to a fully unrestricted setup like Bernardo and Welch (2004)) since traders can anticipate future as well as possibly current trading halts and integrate these anticipations in their choices. Which equilibrium concretely realizes depends on the size of the price limit (Δ) relative to the risk premium ($\gamma\sigma^2$) as well as on how Δ relates to the loss in case of a liquidity shock ($s\mu$). We now discuss each of the three delineated "restricted equilibria" in turn.

A first restricted equilibrium is represented by the blue area in Figure 1 and is labeled *"early-trading equilibrium"*. Within this parameter zone, trading always takes place in the early trading round as the likelihood of the liquidity shock is small and the price limit of the circuit breaker takes intermediate values. In fact, traders do not know their future liquidity needs but trade nevertheless. However, contrary to a fully unrestricted setup like Bernardo and Welch (2004) in which traders only trade because they expect others to trade, in the present case the choice of trading is also driven by the fear that trading can be interrupted. This is particularly visible when we compare the equilibrium number of traded shares in this equilibrium with the number of trades in the fully unrestricted case (see Figure 2). For small values of s, the fear that a large number among the other traders trades is small, therefore trading in the fully unrestricted case is small. However, in the present case featuring a more restrictive circuit breaker, traders also anticipate that in case of a shock the volume pushed into the market will be large and therefore trading will be halted in the late trading round. This induces a higher number of traders to already trade in $t = 0$ as compared to the fully unrestricted case. This number will always be small enough such as to avoid a large price movement and thereby to avoid that the circuit breaker is triggered in $t = 0$. Thus, for small values of s the presence of the circuit breaker induces more traders to sell their share than in the fully unrestricted case. In turn, for larger values of s the fear that a higher number of traders trade at $t = 0$ is greater. As a consequence, in the fully unrestricted case the amount of traded shares increases in a convex way. In contrast, for the present restricted equilibrium, traders know that the circuit breaker will halt trading if too many among them trade in $t = 0$. Therefore the number of traders trading at $t = 0$ is smaller when the circuit breaker interrupts trading at $t = 1$, as compared to the fully unrestricted case. Thus, when the liquidity shock has a larger probability of occurring, the existence of the circuit breaker induces fewer traders to trade in $t = 0$ in comparison to the fully unrestricted case.

A second restricted equilibrium which is labeled *"late-trading equilibrium"* is represented by the red area in Figure 1. Within this zone, the liquidity shock is very likely

and the price limit takes intermediate values. In this parameter zone, trading is always interrupted in $t = 0$ because the selling pressure is strong and pushes the potential transaction price below the price limit of the circuit breaker. In turn, trading only takes place in $t = 1$ in case of a shock.

Finally, a third restricted equilibrium which is labeled *"no-trading equilibrium"* is represented by the green area in Figure 1 in which the price limit is tight and the liquidity shock likely. Within this parameter zone, trading is halted in both trading rounds. Because the liquidity shock is likely, many traders would like to liquidate their share in the early trading round. However, the circuit breaker is defined so tightly that they can never sell.

Please insert Figures 1 and 2 around here.

In a market without an active circuit breaker as in Bernardo and Welch (2004), there is always a market run if the probability of the liquidity shock is larger than 0.5. The same occurs in the case in which there is a circuit breaker on the market with a large price limit (i.e. the "unrestricted equilibrium"). However, in all other equilibria, a market run in $t = 0$ is always prevented from occurring since there is either no trading in the early trading round (in the "no-trading equilibrium" and the "late-trading equilibrium") or the volume is smaller than 1 in the case of the "early-trading equilibrium". The zone in which a market run is avoided due to the presence of a circuit breaker is indicated in Figure 2. Do note, however, that the market run is avoided at the cost that traders cannot trade anymore if they are hit by the liquidity shock at the later period in all equilibria except the "late-trading equilibrium" case. Thus, avoiding the market run through a circuit breaker mechanism and thereby reducing or avoiding socially undesirable trades comes at the cost of also impeding the realization of socially desirable trades. More light will be shed on this issue in the next section featuring a welfare analysis.

3 Welfare Analysis

In the previous section we highlighted that the circuit breaker has two opposite effects on welfare. On the bright side, it might impede undesired trades in the early trading round when traders trade at a cost without knowing whether they truly need to sell their asset. However, the circuit breaker might also hinder trading when traders suffer a liquidity shock in which case their utility loss stems from the impossibility to sell, which is a less desirable feature. In this section we construct a welfare measure in order to determine under which circumstances a restrictive circuit breaker regime is better from a social point of view than a market without such a device. More specifically, welfare

is computed as the sum of all trading gains of those traders who trade in one of the two trading rounds and the utility attached to holding the asset of those traders who do not trade but keep the asset until the last date. Since the market making sector has no profit by definition, it does not enter the calculation of welfare. As such, the computed trader welfare measure comprises overall welfare in this setup.

Our welfare measure depends on the proportion of traders that trade and on the transaction prices at which this trading occurs, both of which hinge on whether the circuit breaker was triggered or not. In the four equilibrium cases presented in the previous subsection, the circuit breaker is triggered in different circumstances and prices as well as the number of trades differ accordingly across the individual cases. Therefore, we will first identify the welfare function for each of the four cases, and compare them in the subsequent part of our welfare analysis.

In the first case, i.e. the "unrestricted equilibrium", the circuit breaker price limit is set very leniently (i.e. $\Delta > \frac{\gamma}{2}\sigma^2$) such that it never prevents trading from occurring. The market functions as if there was no active circuit breaker, and we reach the equilibrium of Bernardo and Welch (2004): a fraction α^* of the traders trades in $t = 0$, and a fraction $(1 - \alpha^*)$ trades at $t = 1$ in case they are hit by a liquidity shock. The welfare level, which we denote by W_1, could then be computed as follows:

$$W_1 = \alpha^*.p_0(\alpha^*) + (1 - \alpha^*).\left[s.p_1(1 - \alpha^*) + (1 - s).\mu\right]$$

Substituting the equilibrium values for α^* and for the transaction prices into this equation yields the level of welfare achieved under our unrestricted equilibrium (or even broader, for a market without an active circuit breaker as in Bernardo and Welch (2004)). Within this unrestricted equilibrium the trading volume takes two different forms hinging on the magnitude of the probability of the liquidity shock s. As a consequence, the welfare function also features a different shape depending on s. In the first case, a liquidity shock is very likely (i.e. $s > \frac{1}{2}$), and a market run occurs (i.e. $\alpha^* = 1$). All traders sell in the early trading round and the welfare level is equal to the price obtained by traders:

$$W_1 = \mu - \frac{\gamma}{2}\sigma^2 \tag{20}$$

In contrast, if the liquidity shock is not likely (i.e. $s \leq \frac{1}{2}$) a market run does not take place and the proportion of traders selling their asset in the early trading round is proportional to the likelihood of the liquidity shock (i.e. $\alpha^* = \frac{s}{1-s}$). In this case, the remaining $(1 - \alpha^*)$ traders sell their share in the late trading round if they are hit by a

shock or receive the payoff of the asset (which is expected to equal μ) at $t = 2$ otherwise. The welfare level is then:

$$W_1 = \mu - \frac{\gamma}{2}\sigma^2 \frac{s}{1-s} \tag{21}$$

Under these conditions, welfare W_1 decreases with the probability s and is at its lowest level when the market run occurs. When the liquidity shock becomes more likely, traders conjecture a higher proportion among them to sell and are therefore incentivized to sell themselves (which is the accelerator effect also evidenced in Bernardo and Welch (2004)). This increases the number of socially undesirable trades and therefore creates a higher welfare loss. The lenient circuit breaker is completely ineffective in preventing this welfare loss from occurring.

In the second case, i.e. the "early-trading equilibrium", the price limit of the circuit breaker takes intermediate values (i.e. $\frac{\gamma}{2}\sigma^2 \geq \Delta > s\mu$). Since the circuit breaker halts trading in the late trading round, traders who did not sell in the early trading round can only obtain a utility gain from holding the asset if they are not hit by the liquidity shock. In this case, welfare is computed as follows:

$$W_2 = \alpha^*.p_0(\alpha^*) + (1-\alpha^*).(1-s).\mu \tag{22}$$

Replacing the equilibrium value for the number of traders liquidating their asset, i.e. $\alpha^* = \frac{2s\mu}{\gamma\sigma^2}$, into this equation yields the welfare level:

$$W_2 = (1-s)\mu \tag{23}$$

Noteworthy, the welfare level is the same as if none of the traders had sold their asset but all had waited until the payoff is paid out. More specifically, within this equilibrium the welfare gain of those traders who have sold in the early trading round if there is a liquidity shock at the later period exactly offsets the welfare loss of those traders who either did not sell in the case there is a shock or who have sold early and the shock did not occur.

In the third case, i.e. the "late-trading equilibrium", the circuit breaker takes intermediate values but the likelihood of the liquidity shock is large (i.e. $\frac{\gamma}{4}\sigma^2 < \Delta \leq s\mu$). A large fraction of traders would like to sell in the early trading round but the volume is large enough to trigger the circuit breaker and trading is postponed to the late trading

round. Traders obtain a utility gain by trading in the late trading round if they suffer a shock or by waiting until the payoff is paid out. Welfare is computed as follows:

$$W_3 = s.p_1(\alpha = 0) + (1 - s).\mu \tag{24}$$

Replacing the price function into this equation yields the welfare level of this equilibrium:

$$W_3 = \mu - \frac{\gamma}{2}\sigma^2 s \tag{25}$$

Here, the unique welfare loss stems from the necessity to sell the asset at a discount when the liquidity shock occurs. In this equilibrium, trading is always desirable because it only occurs when it is driven by a shock.

Finally, in the fourth case developed above, i.e. the "no-trading equilibrium", the price limit of the circuit breaker is defined very tightly ($\Delta \leq Min\left\{\frac{\gamma}{4}\sigma^2, s\mu\right\}$) and trading never occurs in equilibrium. Traders then obtain a utility gain only if they are not hit by the liquidity shock, and welfare is:

$$W_4 = (1 - s).\mu \tag{26}$$

Evidently, in this no-trading case welfare losses result from the inability of traders to sell in the second trading round when the liquidity shock occurs.

Overall, from a social point of view, this setting induces two types of welfare losses for traders: (i) the trading cost due to the price impact and (ii) the realization of potentially undesired trades. Hence, the "first-best equilibrium" yielding maximum welfare would be an equilibrium in which there is no price impact and in which there are no "superfluous" trades. Do note that in the absence of a price impact (i.e. $\gamma = 0$), no market run occurs because traders know that a large selling pressure will not move the price. As such, there is no incentive to trade early and undesired trades are avoided. However, the existence of a price impact (i.e. $\gamma > 0$) in our model conduces to the two delineated types of welfare losses. In this setting, the best welfare outcome is the one in which all traders wait to trade until $t = 1$, and only trade in case they have been hit by the liquidity shock. Thus, a regulator or market succeeding in delaying all trading until the late trading round realizes this outcome avoiding socially undesirable trades and minimizing market impact costs. Within the analysis presented below, we will verify to what extent installing a circuit breaker contributes in reaching this goal. Throughout

this analysis, we will keep this realizable best outcome (given $\gamma > 0$) as a benchmark to which we compare the welfare obtained within the different circuit breaker regimes.

The only equilibrium matching the mentioned benchmark is the "late-trading equilibrium", under which the circuit breaker rule forces traders to wait until the last trading round to sell their asset. Thus, only socially desirable trades are taking place, and traders' utility is always maximal despite the associated trading costs. In turn, when trading occurs at the "wrong" date (i.e. not based on actual liquidity needs but only driven by the fear that others also trade), this is detrimental to social welfare.[15] This result is captured within the following lemma.

Lemma 2 *The "late-trading equilibrium" always produces the highest level of welfare achievable:* $W_3 = \max[W_1, W_2, W_3, W_4]$.

Proof. See Appendix. ■

As demonstrated in the previous subsection, the extent to which this "late-trading equilibrium" is played (and thus the extent to which the first-best welfare outcome can be realized) hinges upon two conditions. First, the price limit Δ must be positioned between two values (i.e. $\frac{\gamma}{2}\sigma^2 \geq \Delta > \frac{\gamma}{4}\sigma^2$) which are completely determined by the characteristics of the asset and of the market. Second, the loss in case of a liquidity shock and when trading is impeded in $t = 1$ (i.e. $s\mu$) must be larger than the price limit of the circuit breaker, which is the case only if $s\mu > \frac{\gamma}{4}\sigma^2$. When the latter condition is not satisfied, the socially optimal circuit breaker cannot be implemented. In contrast, if the latter condition is satisfied, the circuit breaker can always be defined such as to lie within the indicated interval and to be smaller than $s\mu$. This implies that an exchange or a regulator aiming to implement the socially optimal solution may succeed in reaching its set target if it is able to tailor the circuit breaker limit Δ as follows: $s\mu > \Delta > \frac{\gamma}{4}\sigma^2$. Thus, setting the welfare-maximizing circuit breaker level implies accounting (i) for the individual characteristics of the asset, and (ii) for the characteristics of the aggregate shock which may hit the entire trader population. However, evidently, for a given economic setting (i.e. a given shock probability s and market liquidity γ), attaining the optimal circuit breaker design might not be possible for any kind of asset. Even if the design of the circuit breaker is security-specific, the effectiveness of this device depends on market wide variables.

We now turn to an in-depth analysis of the other equilibria to explore what happens to welfare if condition $s\mu > \Delta > \frac{\gamma}{4}\sigma^2$ is not met. More specifically, we compare welfare

[15] Noteworthy, the "early-trading equilibrium" is actually always rendering the same welfare level as the "no-trading equilibrium" (i.e. $W_2 = W_4$, see Proposition 4) which clearly indicates the welfare loss induced by early trading.

levels across the three remaining equilibria (i.e. the "unrestricted equilibrium", the "early-trading equilibrium", and the "no-trading equilibrium") which will allow us to make further statements on how the circuit breaker configuration affects welfare. Do note that since the welfare equations across the different equilibria all do not depend on the circuit breaker level Δ, any comparison to the "unrestricted equilibrium" is completely identical to comparing to the fully unrestricted case in which no circuit breakers are installed at all as in Bernardo and Welch (2004).

First, we find that having a tightly defined circuit breaker on the market may lead to lower or higher welfare as compared to the "unrestricted equilibrium". Remember that welfare gains and losses are not only determined by the number of traders trading but also by the size of the price impact. Accordingly, the results on welfare differ depending on whether the price impact ($\gamma\sigma^2$) is large relative to the expected value of the asset (μ).

First, consider the case in which the price impact is relatively small, i.e. $\gamma\sigma^2 < \mu$ (or $\frac{\gamma\sigma^2}{2\mu} < \frac{1}{2}$ within Figure 2). This condition implies that $1 - \frac{\gamma\sigma^2}{2\mu}$ is larger than $\frac{1}{2}$. We know the "early-trading equilibrium" can only realize for values of $s < \frac{\gamma\sigma^2}{2\mu}$. Furthermore, recall that the number of trades is larger relative to the "unrestricted equilibrium" when s is smaller than $1-\frac{\gamma\sigma^2}{2\mu}$ which is always the case within this parameter configuration. Since the number of undesired trades in $t = 0$ is always larger when the "early-trading equilibrium" occurs, the welfare it generates is always smaller compared to the "unrestricted equilibrium". In turn, in the "no-trading equilibrium", traders do not trade and the unique source of welfare is the final payoff in case there is no liquidity shock. Although this equilibrium does not feature any undesired trades, it also impedes traders to trade when they suffer a liquidity shock which is a utility loss compared to the "unrestricted equilibrium". In fact, within this "unrestricted equilibrium", the welfare loss due to undesired trades in $t = 0$ is small because of the small price impact. Therefore, welfare is always higher in the "unrestricted equilibrium" than in the "no-trading equilibrium". Summarizing, as compared to the "unrestricted equilibrium", for this parameter range the "early-trading equilibrium" and the "no-trading equilibrium" always correspond to a lower welfare level.

Second, when the price impact is larger (i.e. when $\gamma\sigma^2 > \mu$, or $\frac{\gamma\sigma^2}{2\mu} > \frac{1}{2}$ within Figure 2), results change in favor of a restrictive circuit breaker. Consider first the "early-trading equilibrium". We can distinguish two zones. For values of s smaller than $1-\frac{\gamma\sigma^2}{2\mu}$, the number of undesirable trades in $t = 0$ is again larger than in the "unrestricted equilibrium". As a consequence, welfare is smaller. Notice that the previously mentioned conditions imply the absence of a market run on a market without circuit breaker. This case illustrates the advantage of a market where trading is unrestricted. If a liquidity shock is not likely, traders trade little in the unrestricted case because they know that

they will have the possibility to trade in the next period if they need to. Traders also know that the other traders have the same anticipation and thus the total volume in $t = 0$ is small (this is the opposite of the accelerator effect). If a circuit beaker is in place and is defined tight enough so as to block trading in the late trading round, traders anticipate the impossibility to trade at a later period and know that the other traders have the same anticipation. The total volume pushed in the market is then larger, and thereby also the number of potentially undesirable trades which leads to additional welfare losses. However, the equilibrium is now also realizable for values of s larger than $1 - \frac{\gamma\sigma^2}{2\mu}$. When the liquidity shock occurs with a higher probability the number of undesirable trades becomes smaller than in the "unrestricted equilibrium", and therefore welfare is higher in the "early-trading equilibrium". Thus, even though traders trade at the "wrong" date and cannot liquidate in case of a shock, they can achieve a higher total welfare. This occurs even if there is no market run in the "unrestricted equilibrium". Next, consider the "no-trading equilibrium" in which welfare decreases with the probability of a liquidity shock since the shock reduces the utility of the final payoff to zero. In the extreme case, when s tends towards 1, welfare in this "no-trading equilibrium" tends towards zero. Therefore, in this particular case a market run in the "unrestricted equilibrium" is always better from a social point of view than no trading at all despite the large price impact. Moreover, when s is lower than $1 - \frac{\gamma\sigma^2}{2\mu}$, welfare is also higher in the "unrestricted equilibrium" than in the "no-trading equilibrium". This is because the number of undesirable trades is small (due to the convex relationship between α^* and s in the "unrestricted equilibrium") when trading is unrestricted and thus welfare losses are small. On the other hand, the gain if traders can trade in $t = 1$ is also small (due to the large price impact) but is larger than the utility loss related to early trading. In turn, when s takes intermediate values (i.e. $1 - \frac{\gamma\sigma^2}{2\mu} < s < \frac{\gamma\sigma^2}{2\mu}$) welfare is larger in the "no-trading equilibrium". Compared to a situation without trading, unrestricted trading (without a market run) leads to a welfare gain for those traders who liquidate their asset in the late trading round due to the liquidity shock, but it also leads to a welfare loss due to undesired trades in the early trading round. The welfare loss is larger than the welfare gain and thus, no trading is better from a social point of view.

All the delineated results are summarized in the following proposition.

Proposition 4 *The welfare rankings across the four equilibria (i.e. "unrestricted equilibrium" - W_1, the "early-trading equilibrium" - W_2, the "late-trading equilibrium" - W_3, and the "no-trading equilibrium" - W_4) are as follows:*

- *If* $\frac{\gamma\sigma^2}{2\mu} < \frac{1}{2}$*:* $W_3 > W_1 > W_2 = W_4$
- *If* $\frac{\gamma\sigma^2}{2\mu} \geq \frac{1}{2}$*:*

- $0 \leq s < 1 - \frac{\gamma\sigma^2}{2\mu}$: $W_3 > W_1 > W_2 = W_4$
- $1 - \frac{\gamma\sigma^2}{2\mu} \leq s < \frac{\gamma\sigma^2}{2\mu}$: $W_3 > W_2 = W_4 > W_1$
- $\frac{\gamma\sigma^2}{2\mu} \leq s \leq 1$: $W_3 > W_1 > W_2 = W_4$

Proof. See Appendix. ■

Proposition 4 yields further insights on the welfare losses that occur when the best equilibrium (which is the "late-trading equilibrium") is not realized, We now compare the welfare of the three other equilibria to this best equilibrium in order to analyze the nature of the welfare losses incurred relative to this social optimum.

It follows immediately from the first part of the proposition that, compared to the socially best case, the welfare loss when the "unrestricted equilibrium" is realized is smaller than the welfare loss when the "early-trading equilibrium" or the "no-trading equilibrium" are played assuming that the price impact is small. In turn, when the price impact is large, the ranking of the welfare losses depends on the likelihood of the liquidity shock s.

Focusing on the "unrestricted equilibrium" first, this case generates two types of welfare losses as compared to the "late-trading equilibrium". First, since there is always trading in the early trading round, those traders who sold their share incur a utility loss if the shock eventually does not occur. This loss increases with the size of the price impact. Second, if a liquidity shock takes place, those traders who need to trade in the late period suffer a higher price impact than in the socially optimal case.[16] Therefore, when the price impact is small (i.e. $\gamma\sigma^2 < \mu$), the welfare loss with respect to the "late-trading equilibrium" is small.

Next, the "no-trading equilibrium" leads to a unique large utility loss for traders as compared to the "late-trading equilibrium": if they are hit by the liquidity shock, they cannot sell their asset and lose therefore the equivalent of the selling price at that period. This loss increases the smaller the price impact is. If the shock does not occur, traders obtain the same utility than in the "late-trading equilibrium". Thus, the total welfare difference here depends not only on the size of the price impact but also on the probability of the liquidity shock.

Finally, when the "early-trading equilibrium" is implemented, traders suffer two types of utility losses compared to the "late-trading equilibrium". If the liquidity shock occurs, those traders who did not trade in the early trading round lose the complete value of the security μ, while they could have traded and recuperate part of the value in the "late-trading equilibrium". This loss increases the smaller the price impact is. If the liquidity shock does not occur, those traders who traded in the early trading round incur a utility loss compared to the "late-trading equilibrium" because they pay the

[16] See the discussion in Subsection 2.1 on equilibrium prices for the intuition.

price impact. These two types of utility losses are differently affected by the size of the price impact. Therefore, here again, the size of the total welfare loss relative to the "late-trading equilibrium" depends not only on the price impact but also on the probability of the liquidity shock. Since the welfare here is the same as in the "no-trading equilibrium" (see Proposition 4), the total welfare loss relative to the "late-trading equilibrium" is also higher when the price impact becomes smaller.

In conclusion, if the price impact is small (i.e. $\gamma\sigma^2 < \mu$, or part 1 of Proposition 4), the welfare loss generated by the "unrestricted equilibrium" is small while the welfare loss generated by the "early-trading equilibrium" and by the "no-trading equilibrium" is high. In turn, consider the case in which the price impact is large (i.e. $\gamma\sigma^2 > \mu$, or part 2 of Proposition 4). From above we know that the welfare loss induced by the "unrestricted equilibrium" always increases with the price impact. In contrast, the welfare loss generated by the "early-trade equilibrium" and the "no-trade equilibrium" decreases with the price impact in the case in which there is a liquidity shock. Therefore, when the probability of a liquidity shock is small (i.e. when $s < 1-\frac{\gamma\sigma^2}{2\mu}$), the "unrestricted equilibrium" outranks the "early-trade equilibrium" and the "no-trade equilibrium" as it yields a smaller welfare loss as compared to the socially best equilibrium. If, in turn, the probability of the shock takes intermediate values (i.e. $1 - \frac{\gamma\sigma^2}{2\mu} \leq s < \frac{\gamma\sigma^2}{2\mu}$), the "unrestricted equilibrium" performs worse. Finally, when the liquidity shock is very likely, the welfare of traders if the event they cannot trade is close to zero. Therefore, when a market run occurs, the welfare loss compared to the "late-trading equilibrium" is smaller.

4 Concluding Remarks

In the recent volatile times upon financial markets, exemplified by the 2010 flash crash, across the globe regulators and market operators have developed a renewed interest in an "old" instrument to curb market volatility: "circuit breakers". These trading halt mechanisms are currently actively updated/reinstalled to face modern market circumstances, restore stability and thereby prevent vicious liquidity crises from taking place. Surprisingly, to our knowledge, no research has been conducted on the usefulness of circuit breakers to attain this goal. This paper aims to fill this gap in the academic literature, and contribute to the ongoing debate between regulators, market operators and market practitioners on the validity of circuit breakers in doing so. To this extent, we construct a liquidity crisis setting in which traders fail to coordinate their actions and trade massively although they should refrain from it (see Shiller (1987)). More specifically, our framework of analysis is a financial market translation of the seminal model of bank runs developed by Diamond and Dybvig (1983) in the spirit of Bernardo and Welch (2004). Within this translated version of the model, traders might perform

a collective early market run out of fear of not being able to satisfy potential urgent liquidity needs in the future. We analyze how installing a circuit breaker within this setting may alleviate this coordination failure and thereby prevent such a market run (and the associated welfare losses) from occurring.

The effectiveness of the circuit breaker is shown to hinge upon two factors. A first driving factor is how the price limit at which trading is interrupted is defined relative to the price impact traders incur upon selling. A second driving factor is the expected loss due to the possibility of a liquidity shock. Bearing this in mind, we distinguish four separate equilibrium cases in our analysis depending on the parameter regions where the price limit of the circuit breaker is set. In a first *"unrestricted equilibrium"* a very lenient price limit is set, and trading is never impeded. Under these conditions, installing a circuit breaker is totally ineffective in preventing a market run from occurring. A second *"early-trading equilibrium"* realizes when the applied price limit is tight relative to the price impact and when the expected loss in case of a liquidity shock is small. Under these circumstances, traders rationally anticipate that trading is halted if they suffer a liquidity shock. Thus, traders might be induced to trade more in the early trading round as compared to the unrestricted case. Next, a third *"late-trading equilibrium"* arises when the price limit takes intermediate values and the expected loss related to a liquidity shock is large. In this case, large early selling pressure triggers the circuit breaker. As such, trading is only possible in the second trading round and traders only realize trades if these are motivated by liquidity reasons. Finally, a fourth *"no-trading equilibrium"* occurs when the circuit breaker is set very tightly and the expected loss due to the liquidity shock is large. Under these circumstances, trading is halted in both trading rounds.

Overall, the "late-trading equilibrium" appears optimal as the circuit breaker exogenously forces agents to delay trading until they are aware of their true liquidity needs. As such, within this equilibrium the circuit breaker effectively resolves the existing coordination failure among traders and leads to the highest possible level of social welfare. However, this equilibrium is not always realizable as its implementation entails specific economic conditions are met. When this is not the case (and thus when one of the other three equilibria is played), the circuit breaker is shown to actually prevent the realization of socially desirable trades and stimulate the occurrence of socially undesirable trades.

Finally, in future research we plan to investigate to what extent circuit breakers are successful in reaching their intended target of lowering price volatility and thus increasing market stability. In doing so, the trade-off between volatility and welfare will be analyzed. Furthermore, we intend to verify how introducing uncertainty on the circuit breaker price limit and sequential trading affects our results.

Appendix: Proofs

Proof of Proposition 1.

By assumption $\Delta > \frac{\gamma}{2}\sigma^2$ holds, which is equivalent to $\frac{2\Delta}{\gamma\sigma^2} > 1$. The circuit breaker is triggered if $\alpha > \widetilde{\alpha} = \frac{2\Delta}{\gamma\sigma^2}$. Thus $\widetilde{\alpha} > 1$. Since $\alpha \in [0, 1]$, the circuit breaker is never triggered.

Q.e.d. ■

Proof of Proposition 2.

The indifference condition of traders is:

$$F(\alpha^*) = p_0(\alpha^*) - (1 - s)\mu = 0$$

With $p_0(\alpha^*) = \mu - \frac{\gamma}{2}\sigma^2\alpha^*$, the equality becomes:

$$\mu - \frac{\gamma}{2}\sigma^2\alpha^* = (1 - s)\mu$$

$$\Leftrightarrow \frac{\gamma}{2}\sigma^2\alpha^* = s\mu$$

$$\Rightarrow \alpha^* = 2\frac{s\mu}{\gamma\sigma^2}$$

To be a feasible equilibrium:

$$\alpha^* \leq \widetilde{\alpha} \Leftrightarrow 2\frac{s\mu}{\gamma\sigma^2} \leq \frac{2\Delta}{\gamma\sigma^2} \Leftrightarrow s\mu \leq \Delta$$

If $s\mu > \Delta$, traders cannot trade the amount α^* in $t = 0$ because it launches the circuit breaker. They have two possible strategies: or some of them refrain from trading so as to reduce α enough, or a total amount of α^* is submitted but traders know that they can trade in $t = 1$ if they are hit by a shock.

If a trader refrains from trading today, his expected utility is: $s * 0 + (1 - s)\mu$. If the same traders submits his order, and trading is halted in $t = 0$, his expected utility is:

$$sp_1(1) + (1 - s)\mu$$

We know that

$$sp_1(1) + (1 - s)\mu > (1 - s)\mu$$

therefore an individual trader never optimally refrains from trading.

Q.e.d. ■

Proof of Proposition 3.

For trading to take place:

$$\alpha^* = 2\frac{s\mu}{\gamma\sigma^2} \leq \frac{2\Delta}{\gamma\sigma^2} \Leftrightarrow s\mu \leq \Delta$$

Therefore: $s\mu \leq \Delta \leq \frac{\gamma}{4}\sigma^2$.

Here, $s\mu$ has an upper bound at $\frac{\gamma}{4}\sigma^2$ which bounds also α^* whereas in the previous case the upper bound for $s\mu$ is $\frac{\gamma}{2}\sigma^2$. All trading volumes $\alpha^*(s\mu)$ with $s\mu \in \left[\frac{\gamma}{4}\sigma^2, \frac{\gamma}{2}\sigma^2\right]$ are possible in the previous case while they are not possible in this case.

Q.e.d. ■

Proof of Lemma 2.

In order to show that the "late-trading equilibrium" leads the welfare ranking, we compare the welfare in this case to the welfare obtained in all other cases.

<u>Part 1</u>

Welfare in the "late-trading equilibrium" and the "no-trading equilibrium" are $W_3 = \mu - \frac{\gamma}{2}\sigma^2 s$ and $W_4 = (1-s)\mu$, respectively. To show that $W_3 > W_4$ for all $s \in [0,1]$, we proceed by contradiction.

Assume that $W_3 < W_4$. This is equivalent to:

$$\mu - \frac{\gamma}{2}\sigma^2 s < (1-s)\mu \Leftrightarrow 0 > \mu - \frac{\gamma}{2}\sigma^2$$

When the "late-trading equilibrium" is realized and $s = 1$, the transaction price in $t = 1$ is $\mu - \frac{\gamma}{2}\sigma^2$. The price must be positive, so that the previous inequality does not hold. Thus, for any s, $W_3 > W_4$. Since $W_4 = W_2$, it follows that $W_3 > W_2$.

<u>Part 2</u>

To show that $W_3 > W_1$ for all $s \in [0,1]$, we use the same procedure.

The "unrestricted equilibrium" contains two cases: welfare is $\mu - \frac{\gamma}{2}\sigma^2\frac{s}{1-s}$ if $s < \frac{1}{2}$ and $\mu - \frac{\gamma}{2}\sigma^2$ if $s \geq \frac{1}{2}$. We distinguish these two cases in what follows.

- Case 1: $s < \frac{1}{2}$

  Assume that $W_3 < W_1$. This is equivalent to:

$$\mu - \frac{\gamma}{2}\sigma^2 s < \mu - \frac{\gamma}{2}\sigma^2\frac{s}{1-s} \Leftrightarrow s < 0$$

This inequality is never true since s is always weakly positive. Thus, $W_3 > W_1$ when $s < \frac{1}{2}$.

- Case 2: $s \geq \frac{1}{2}$

  $W_3 < W_1$ is equivalent to:

$$\mu - \frac{\gamma}{2}\sigma^2 s < \mu - \frac{\gamma}{2}\sigma^2 \Leftrightarrow s > 1$$

  This inequality is not true since s is always weekly smaller than 1. Therefore $W_3 > W_1$ for $s \geq \frac{1}{2}$.

We have demonstrated that W_3 is the highest achievable level of welfare.

Q.e.d. ■

Proof of Proposition 4.

From the proof of Lemma 2 we know that W_3 is larger than all other welfare levels. We compare W_1 and W_2 in order to rank all welfare levels.

Assume that $W_1 < W_2$ and $s < \frac{1}{2}$. This is equivalent to:

$$\mu - \frac{\gamma}{2}\sigma^2 \frac{s}{1-s} < (1-s)\mu \Leftrightarrow 1 - \frac{\gamma\sigma^2}{2\mu} < s$$

If $1 - \frac{\gamma\sigma^2}{2\mu} < \frac{1}{2}$, the level of welfare is larger in the "early-trading equilibrium" if and only if $1 - \frac{\gamma\sigma^2}{2\mu} < s < \frac{1}{2}$. If $1 - \frac{\gamma\sigma^2}{2\mu} > s$ welfare is higher in the "unrestricted equilibrium". If $1 - \frac{\gamma\sigma^2}{2\mu} \geq \frac{1}{2}$, welfare is always higher in the "unrestricted equilibrium".

Assume now that $W_1 < W_2$ and $s \geq \frac{1}{2}$. This is equivalent to:

$$\mu - \frac{\gamma}{2}\sigma^2 < (1-s)\mu \Leftrightarrow \frac{\gamma\sigma^2}{2\mu} > s$$

The "early-trading equilibrium" can only be realized if $\frac{\gamma\sigma^2}{2\mu} > s$. Therefore, if $\frac{\gamma\sigma^2}{2\mu} > \frac{1}{2}$, welfare is always higher in the "early-trading equilibrium". If $\frac{\gamma\sigma^2}{2\mu} < \frac{1}{2}$, the "early-trading equilibrium" does not realize for $s \geq \frac{1}{2}$.

Summarizing, we have shown the following. If $1 - \frac{\gamma\sigma^2}{2\mu} < \frac{1}{2}$, $W_2 > W_1$ if and only if $s > 1 - \frac{\gamma\sigma^2}{2\mu}$. Otherwise $W_2 < W_1$. If $1 - \frac{\gamma\sigma^2}{2\mu} > \frac{1}{2}$, it follows that $\frac{\gamma\sigma^2}{2\mu} < \frac{1}{2}$ and $W_2 < W_1$ for all s.

The welfare levels in the "unrestricted equilibrium" and the "no-trading equilibrium" are the same. However, the equilibria realize in different parameter regions.

- If $s < \frac{1}{2}$, $W_1 < W_4$ is true if and only if $1 - \frac{\gamma\sigma^2}{2\mu} < \frac{1}{2}$ and $1 - \frac{\gamma\sigma^2}{2\mu} < s$. If $1 - \frac{\gamma\sigma^2}{2\mu} \geq \frac{1}{2}$, $1 - \frac{\gamma\sigma^2}{2\mu} > s$ and thus $W_1 > W_4$.
- If $s \geq \frac{1}{2}$, $W_1 < W_4$ is true if and only if $\frac{\gamma\sigma^2}{2\mu} > s$. Also, $\frac{\gamma\sigma^2}{2\mu} < 1$ because the transaction price in $t = 1$ in case of a liquidity shock ("unrestricted equilibrium") must be positive by assumption ($\mu - \frac{\gamma\sigma^2}{2} > 0$). The "no-trading equilibrium" can occur for any values between 0 and 1 for s. Therefore, when $\frac{\gamma\sigma^2}{2\mu} < s$, $W_1 > W_4$.

Summarizing, there are two cases in which W_1 and W_4 are ranked differently:

- for $\frac{\gamma\sigma^2}{2\mu} > \frac{1}{2} \Leftrightarrow 1 - \frac{\gamma\sigma^2}{2\mu} < \frac{1}{2}$:
  - $0 < s < 1 - \frac{\gamma\sigma^2}{2\mu}$: $W_1 > W_4$
  - $1 - \frac{\gamma\sigma^2}{2\mu} < s < \frac{\gamma\sigma^2}{2\mu}$: $W_1 < W_4$
  - $\frac{\gamma\sigma^2}{2\mu} < s < 1$: $W_1 > W_4$
- for $\frac{\gamma\sigma^2}{2\mu} < \frac{1}{2} \Leftrightarrow 1 - \frac{\gamma\sigma^2}{2\mu} > \frac{1}{2}$, $W_1 > W_4$ for all s.

Putting together the different results yields the result delineated in the proposition.

Q.e.d. ■

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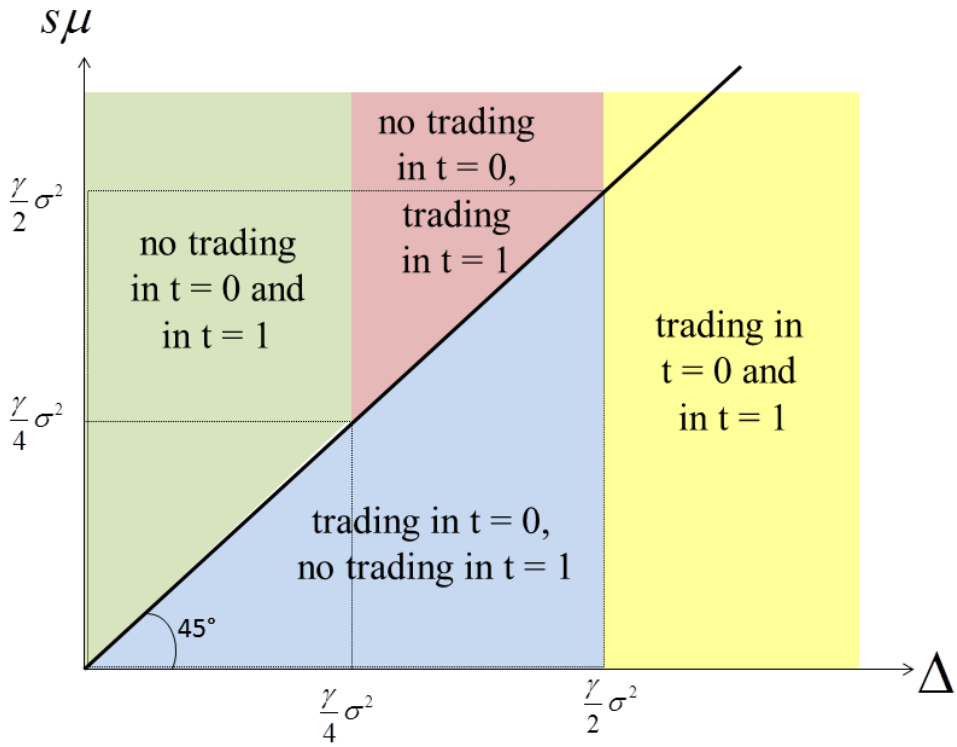
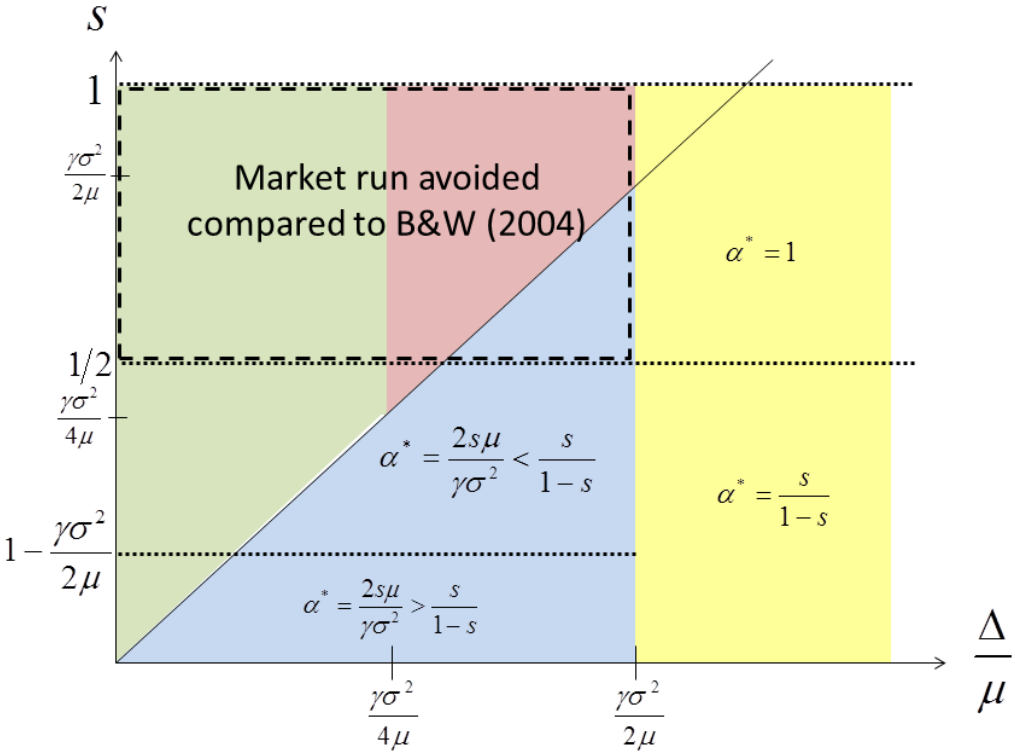
Figure 1: Positioning of the Equilibria



Note: This figure illustrates the four equilibria described in Subsection 2.4. The yellow area corresponds to the "unrestricted equilibrium", the blue area corresponds to the "early-trading equilibrium", the red area corresponds to the "late-trading equilibrium" and the green area corresponds to the "no-trading equilibrium".

Figure 2: Positioning of the Equilibria (ctd.)



Note: This figure displays the trading volumes in the early trading round ($t = 0$) in the different equilibria. We represent the case in which $\frac{\gamma\sigma^2}{2\mu}$ is larger than $\frac{1}{2}$. In the yellow area (the "unrestricted equilibrium") a market run occurs (i.e. $\alpha^* = 1$) if the probability of a shock s is greater than $\frac{1}{2}$, and $\alpha^* = \frac{s}{1-s}$ if $s < \frac{1}{2}$. In the blue area (the "early-trading equilibrium") the volume is equal to $\frac{2s\mu}{\gamma\sigma^2}$. This volume is larger than in the "unrestricted equilibrium" when $s < 1 - \left(\frac{\gamma\sigma^2}{2\mu}\right)$ and it is smaller than in the "unrestricted equilibrium" when s lies above this threshold. In the other two equilibria trading does not take place in $t = 0$. The area delimitated by the dashed line represents the parameter region in which a market run is avoided with a circuit breaker in place compared to the fully unrestricted case of Bernardo and Welch (2004).